Measuring the Output and Prices of the Lottery Sector

Kam Yu
Lakehead University

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Motivation

- SNA 1993 - Direct measurement of government outputs, some countries have implemented the recommendations

- Game of chance - not in the CPI

- Survey of Household Spending (2001) – government lottery:
  - Household participation = 62%
  - Average expenditure per household = \$257
  - Expenditure share = 0.3%

- Revenue reported by the government: 3 times more than SHS

- Current SNA: Input method

- Recent development in economics of risk and uncertainty can provide direct measurement

Economics of Uncertainty

Expected Utility Hypothesis:

- $U = \sum_{i=1}^{N} u_i P_i$
- Implies that a risk-averse expected utility maximizer will never buy lottery tickets

Nonexpected Utility Theory

- First order risk aversion: risk premium for small gamble proportional to standard deviation of the gamble

- Second order risk aversion: proportional to variance (EUH)

- Implicit expected utility:

$$\sum_{i=1}^{N} P_i \phi_u(x_i) - \phi_u(u) = 0$$

Applications

- Consumption-saving analysis, Chew & Epstein (1990): An extension to the permanent income hypothesis

- Equity premium puzzle, Epstein & Zin (1990):
  - Historical risk premium in U.S. = 6.2%
  - Risk premium using NUT = 2%

- Intertemporal behaviour of consumption and asset return, Epstein & Zin (1991): An extension to CAPM

- Measuring outputs in insurance and gambling industries, Diewert (1992,1995): Apply IUT to simple cases

Lotto 6/49

Interprovincial Lottery Corporation – established by the provincial lottery organizations in 1976 to operate joint lottery games across Canada.

Each provincial organization is individually responsible for marketing the national games within its own jurisdiction, and revenues are returned to each province in proportion to generated sales.

Prize	Combination	Chance of Winning (1 in)
Jackpot	All 6 numbers	13,983,816
Second	5 numbers + bonus	2,330,636
Third	5 numbers	55,492
Fourth	4 numbers	1,032
Fifth	3 numbers	57

Payout Prizes

- Prize Fund: 45% of sales
- Pools Fund: Prize Fund minus all 5th prize payout ($10)
- Jackpot: 50% of pool fund
- Second: 15% of pool fund
- Third: 12% of pool fund
- Fourth: 23% of pool fund

Factor Costs

- Retail commissions = 6.3%
- Lottery Corporation operating expenses = 5.6%
- Ticket printing = 1.4%
- Total factor costs = 13.3%

Notation

$$
\begin{aligned}
\phi_i &= \text{chance of a single ticket winning the} \\
&\quad\ i\text{th prize, } i = 1, \ldots, 5 \\
w &= \text{wager (real)} \\
P_i &= \text{total probability of winning the} \\
&\quad\ i\text{th prize, } i = 1, \ldots, 5 \\
P_6 &= \text{prob. of not winning} \\
x_i &= \text{state contingent consumption,} \\
&\quad\ i = 1, \ldots, 6 \\
y &= \text{real disposable income} \\
R_i &= i\text{th payout prize, } i = 1, \ldots, 6, R_6 = 0
\end{aligned}
$$

Expected value of a Lotto 6/49 ticket $= \sum_{i=1}^{5} \phi_i R_i$

$$P_i = w\phi_i, \quad i = 1, \ldots, 5$$

$$P_6 = 1 - \sum_{i=1}^{5} P_i = 1 - w \sum_{i=1}^{5} \phi_i$$

$$x_i = y + R_i - w, \quad i = 1, \ldots, 6$$

How to buy Lotto 6/49

- There are unpopular numbers. They can be estimated using the number of winning tickets at each draw.

- Expected value of a ticket with the 6 most unpopular number = \$1.11

- Expected value of a ticket with the 6 most popular number = \$0.22

- U.K. unpopular numbers: 36, 41, 46, 47, 48, 49

Application of IUT to Lotteries

Utility: $u = F(x_1, \ldots, x_6, P_1, \ldots, P_6)$
such that

$$\sum_{i=1}^{6} P_i \gamma(x_i/u) - \gamma(1) = 0 \tag{1}$$

Use a kinked function for γ to model first order risk-averseness:

$$\gamma(z) = \begin{cases} \alpha + (1-\alpha)z^\beta & \text{for } z \geq 1 \\ 1 - \alpha + \alpha z^\beta & \text{for } z < 1 \end{cases} \tag{2}$$

with parameter restrictions

$$0 < \alpha < 1/2, \quad \beta < 1, \quad \beta \neq 0.$$

Use (2) in (1) and solve for u:

$$u = \left[\frac{(1-\alpha)w \sum_{i=1}^{5} \phi_i (y + R_i - w)^\beta + \alpha(1 - w \sum_{i=1}^{5} \phi_i)(y - w)^\beta}{\alpha + (1 - 2\alpha) w \sum_{i=1}^{5} \phi_i} \right]^{1/\beta} \tag{3}$$

Utility maximization:

$$\max_w \{u : 0 \leq w \leq y\}$$

Regression equation can be derived from the first-order condition, so that α and β can be estimated.

Output of Lotto 6/49:

- Use estimated values of α and β to calculate utility u^{*t} in period t using equation (3)

- Utility u^{0t} as if there is no lottery is equal to y^t

- Real output of Lotto 6/49, $Q^t = u^{*t} - u^{0t}$

- Implicit price, $P^t = w^t/Q^t$

Data:

- Data source: LotteryCanada.com

- Sales volume and payout prizes from 1/11/97 to 3/11/01 (419 observations)

- Highest Jackpot won on 15/11/97, with $15 million

Regression results:

	COEFFICIENT	ST. ERROR	T-RATIO
α	0.10458	0.31648E-02	33.045
β	-31.986	5.9527	-5.3734

With these estimated values of α and β the output and price can be computed.

Price elasticity of demand, $\epsilon = -0.67$

Farrell and Walker (1999):
U.K. cross-sectional data: $\epsilon = -0.76$

Forrest, Gulley, and Simmons (2000):
Two-stage OLS: U.K. $\epsilon = -0.66$

Beenstock and Haitovsky (2001):
Time series data from Israel: $\epsilon = -0.65$