MEASURING MULTI-FACTOR PRODUCTIVITY WHEN RATES OF RETURN ARE EXOGENOUS

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Abstract

This paper discusses the computation of capital services measures with user cost expressions that employ exogenous rates of return, as well as expected depreciation and expected asset price changes. One consequence of this formulation is that total capital remuneration does not necessarily equal non-labour income as given by the national accounts. The paper proposes several interpretations of this discrepancy, discusses implications for MFP measures and growth accounting and puts forward one preferred productivity measure. The methods are implemented empirically for four OECD countries.

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1. Introduction: gross operating surplus and the remuneration of capital

Official statistics do not normally provide direct observations on the price and volume of capital services. What is available from the national accounts is a measure of gross operating surplus, i.e., a measure of business profits from normal business activity, including mixed income, the income of self-employed persons. Thus, national accounts provide the researcher with data according to the following accounting identity:

(1) $$P \cdot Q = wL + GOS$$

where $P \cdot Q$ is the sum of current-price output in the economy, with $P = [P_1, P_2, ...P_M]$ the vector of prices and $Q = [Q_1, Q_2, ...Q_M]$ the vector of quantities of output. To simplify notation, we use $P \cdot Q \equiv \sum_{i=1}^{M} P_i Q_i$ for the inner product between P and Q. Output is measured as value-added, and prices are basic prices, i.e., they exclude all taxes on products and include subsidies on products. wL is the remuneration of labour with a wage component w and a volume component L, and GOS is the gross operating surplus. For simplicity, it will be assumed here that mixed income is either zero or has been split up between the labour and the GOS component. Thus, (1) re-states the production and the income side of the national accounts.

The national accounts provide no indication as to exactly which factor of production is remunerated through GOS. Fixed assets are certainly among them but they are not necessarily the only ones. The business literature offers a wealth of discussions about the importance of intangible assets, and there are good reasons to argue that such assets account at least for part of GOS. While this may appear a minor point, it puts in question an assumption routinely made by analysts of productivity and growth, namely that GOS exactly represents the remuneration of the fixed assets recognised in the System of National Accounts (SNA), or the value of the services of these assets. Assume that $u^* = [u_1^*, u_2^*, ...u_N^*]$ is a vector of user costs for N types of capital services and $K = [K_1, K_2, ...K_N]$ is the corresponding vector of the quantities of capital services flows, then the assumption is typically made that $u^* \cdot K = GOS$, i.e., gross operating surplus corresponds exactly to the value of capital services derived from fixed assets (as before, we have used the shorthand $u^* \cdot K$ to denote the product of price and quantity vectors: $u^* \cdot K \equiv \sum_{i=1}^{N} u_i^* K_i$).

In other words, it is assumed that remuneration of capital services exactly exhausts gross operating surplus. Empirically, the equality $u^* \cdot K = GOS$ is obtained by choosing an appropriate value for the net rate of return to assets, which is part of the user costs[1]. Thus, the rate of return has been fixed *endogenously*.

This assumption is convenient because it is consistent with competitive behaviour on product and factor markets and a production process that exhibits constant returns to scale. Conditions such as these are required if (1) should be re-written as

$$(2) \qquad P \cdot Q = wL + u^{*} \cdot K$$

i.e., if gross operating surplus should correspond exactly to the remuneration of fixed assets. Counter to claims sometimes made in the literature (Hsieh 2002 is a recent example) (2) is *not* an accounting identity. For example, for (2) to hold,

- the set of assets $\{K_1, K_2, \ldots K_N\}$ has to be complete in the sense that all assets are observed by the statistician who compiles the national accounts;
- the ex-post rate of return on each asset (implicitly observed by the national accountant as part of GOS) equals its ex-ante rate return, the economically relevant part in the user costs of capital services;
- there has to be absence of residual profits (or losses) that may arise in the presence of market power, under non-constant returns to scale or with publicly available capital assets.

It is not immediately obvious that such conditions hold empirically. What happens in the more general case? Suppose that the net rates of return are not chosen endogenously but on the basis of other economic information such as market interest rates. In this case, i.e., under exogenous rates of return to capital, there will be an independent observation for the price and the value of capital services and there is no guarantee that the sum of labour remuneration and capital remuneration equals total revenues or total value added at current prices. This raises the question how under such circumstances, indices of multi-factor productivity (MFP) should be computed and how growth accounting exercises should be carried out. The present paper discusses some of the options, and identifies the theoretical assumptions that have to be made to implement non-parametric measures of productivity growth that aim at capturing technical change. These assumptions tend to be restrictive and as a way out we expresses preference for a simple MFP measure that – while it cannot pretend at capturing technical change – is readily implemented and does not impose strong a-priori conditions on producer behaviour and production technology. Few things are new among the ideas expressed below except perhaps that they have been brought together in one place. Much of the material builds on Jorgenson and Griliches (1967), Fuss and McFadden (eds.) (1978), Diewert and Nakamura (2005) and Harper et al. (1989).

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[1] In a simple continuous-time formulation, the user costs or rental price of an asset (Jorgenson and Griliches1967) is given by $u^i = q^i\left(r + \delta^i - d\ln q^i / dt\right)$ where $q^i(t)$ is the purchase price of a new asset of type i, $r(t)$ is the net rate of return, δ^i is a rate of depreciation, $d\ln q^i / dt$ is rate of change of q^i .

2. Why GOS may differ from remuneration of capital

This paper considers thus a more general formulation of the income-production relationship (2) by allowing for independent measures of capital remuneration, $u = [u_1, u_2, ... u_N]$. Under these circumstances, equation (2) becomes $P \cdot Q - wL = GOS = u \cdot K + M$. Thus there is a term M that corresponds to the difference between the observed or imputed remuneration of assets $u \cdot K$ and the value of gross operating surplus or non-labour input. If one uses $C \equiv wL + u \cdot K$ as shorthand for observed factor payments, one could also say that gross operating surplus is split into one component that reflects observable factor remuneration and another, residual one with several possible interpretations. In principle, there is no restriction on the sign of M. However, a negative term over an extended period of time would imply sustained losses of is economically implausible and shall not be considered further. Consequently, in what follows, non-negativity of M is assumed[2].

We propose several interpretations of M. The first one is the existence of mark-ups of prices over total costs. Such mark-ups may exist if output markets are not fully competitive so that M reflects a monopoly rent. These may be short-run mark-ups that disappear over longer periods. Mark-ups could also reflect Schumpeterian growth patterns where such mark-ups constitute the incentives for entrepreneurial activity and behaviour. Finally, mark-ups could arise in industries where long gestation periods of investment are prevalent.

Models of short-run disequilibrium over the business cycle provide further theoretical justifications for the existence of M. One explanation is time-varying capacity utilisation, dealt with by Berndt and Fuss (1986) and Hulten (1986). Cyclicality of productivity measures and their relation to technical change are dealt with by Basu and Kimball (1997) who find strong effects of variable utilisation of capacity on measures of productivity. Such effects could be picked up in the movements of M.

The second interpretation of M is the existence of pure profits as a consequence of the presence of decreasing returns to scale, combined with marginal cost pricing of every output. Alternatively, M could be positive in the presence of increasing returns to scale and a positive mark-up over marginal costs. The size of M will then depend on the degree of competition in output markets: free market entry and competition will drive mark-ups and prices to a level where total revenues just cover total costs, implying M=0. If markets are not fully competitive, however, or in transition phases, M may well be positive. The Lucas-Romer model of endogenous growth (Romer 1990) could provide another theoretical underpinning for such a situation: at the firm level, returns to scale are constant, at the aggregate level they are increasing due to externalities.

[2] For the four countries reviewed in the empirical part of this paper (Canada, France, Japan , United States) M is positive in nearly all years during the period 1980-2002. If M were negative over an extended periods of time, this would cast doubt on the measures for the remuneration of capital, in particular the choice of the exogenous rates of return.

The third interpretation puts forward the existence of unobserved inputs and essentially describes a statistical phenomenon. It arises when not all capital inputs that give rise to operating surplus (itself estimated residually in the national accounts as non-labour income) are recognised in the national accounts. In contrast to the second interpretation, we would expect M to remain positive even in the longer run because true total costs are higher than what appears from observed assets and M would just about cover these hidden costs. Non-observed inputs and their link to measured MFP growth and technical change have been investigated by Basu et al. (2003): the authors introduce unobserved intangible organisational capital that they take as complementary to observed investment in information technology[3].

3. Production technology and producer behaviour

To introduce production technology, we call Z(t) a feasible set of inputs and outputs in period t. We further assume that there is a total cost function TC that shows minimum costs of production, given a vector Q of M outputs $[Q_1, Q_2, \ldots Q_M]$ and given a set of input prices. Inputs comprise labour L, N different types of observed capital services $K_1, \ldots K_N$ and one non-observable asset D. The corresponding prices are w, the wage rate, $u = [u_1, u_2, \ldots u_M]$, the user costs of capital, and ϕ, the price of the unobserved input D. The total cost function is then defined as in (3)

$$\text{(3)} \qquad TC[Q, w, u, \phi, t] = \min_{L,K,D} \{wL + u \cdot K + \phi D : (Q, L, K, D) \text{ belongs to } Z(t)\}$$

The cost function is linearly homogenous in input prices and non-decreasing but not necessarily linear homogenous in the vector of outputs $[Q_1, \ldots Q_M]$. Thus, there is no assumption about constant returns to scale. However, producers are assumed to minimise total cost, so that actual costs equal minimum costs ($wL + u \cdot K + \phi D = TC(Q, w, u, \phi, t)$). Further, producers face competitive factor markets and Shephard's (1953) condition for optimality of factor input applies:

$$\text{(4a)} \qquad L = \left(\frac{\partial TC}{\partial w}\right);$$

$$\text{(4b)} \qquad K_i = \left(\frac{\partial TC}{\partial u_i}\right); \; i=1,\ldots N;$$

$$\text{(4c)} \qquad D = \left(\frac{\partial TC}{\partial \phi}\right).$$

On the output side, we allow for imperfect product markets and only state that output prices are proportional to marginal costs. No explicit assumption is made about the kind of imperfect competition that prevails or whether producers are profit maximising or not.

[3] Unlike the present model, however, theirs is a general equilibrium setting that accounts not only for unobserved intangible inputs but also for the unobserved production of these investment goods.

All that is needed is a relationship between prices and marginal costs so that if the price of output i is P_i and if $1 \leq 1/\mu_i$ is a product-specific, time-varying mark-up factor, producer behaviour on the output side is described by

(5) $P_i \mu_i = \partial TC / \partial Q_i \quad i = 1,...M$.

Next, we follow the literature (e.g., Panzar 1989) and measure the local elasticity of cost with respect to scale as

(6) $$\varepsilon \equiv \sum_{i=1}^{M} \frac{\partial \ln TC}{\partial \ln Q_i} .$$

$\varepsilon > 0$ indicates the percentage change in total cost for a given percentage change in all outputs and its inverse can readily be interpreted as a measure of local returns to scale: $\varepsilon > 1$, for example implies that a one percent rise in the quantity of all outputs increases total costs by more than one percent and this is tantamount to a situation of decreasing returns to scale. Similarly, $\varepsilon < 1$ and $\varepsilon = 1$ correspond to increasing and constant returns to scale respectively[4].

Given (5), the measure of scale elasticity can be further transformed:

(7) $$\begin{aligned} \varepsilon &\equiv \sum_{i=1}^{M} \frac{\partial \ln TC}{\partial \ln Q_i} = \sum_{i=1}^{M} \frac{P_i Q_i \mu_i}{TC} \\ &= \sum_{i=1}^{M} \frac{P_i Q_i \mu_i}{P \cdot Q} \frac{P \cdot Q}{TC} \\ &= \mu \frac{P \cdot Q}{TC} \quad \text{where } \mu \equiv \sum_{i=1}^{M} \frac{P_i Q_i}{P \cdot Q} \mu_i \end{aligned}$$

In (7) μ is the economy-wide inverted average mark-up factor – a weighted average of industry-specific mark-ups with simple output shares as weights. Expression (7) reads as $P \cdot Q = (\varepsilon / \mu) TC$. Thus, the value of total output revenues equals total costs, adjusted by a mark-up factor $1/\mu$, itself a weighted average of the product-specific mark-up factors in the economy, and by ε, the parameter for the scale elasticity.

(7) can now be combined with the national accounts information mentioned earlier. In particular, it was pointed out that gross operating surplus is defined as the difference between the value of output and labour income: $GOS = P \cdot Q - wL$. Using the result $P \cdot Q = (\varepsilon / \mu) TC$, one obtains $GOS = TC\varepsilon / \mu - wL$. Also, the difference between GOS and observed capital income has been labelled M earlier on: $M = GOS - u \cdot K$. Inserting the

4 As we operate with a multi-product cost function, a distinction needs to be made between general economies of scale and product-specific economies of scale. The former – treated here – deals with changes in costs when all outputs are changed by the same proportion. The latter deals with changes in costs as one particular output is increased while holding all other outputs constant. We shall not deal with the latter form of economies of scale here but refer to Panzar (1989) for a full discussion.

expression for GOS and taking into account the definition of TC produces a relation for M that can readily be interpreted:

$$\text{(8a)} \qquad M = \left(\frac{\varepsilon}{\mu} - 1\right) TC + \phi D \quad \text{or}$$

$$\text{(8b)} \qquad M = P \cdot Q \left(1 - \frac{\mu}{\varepsilon} + \frac{\phi D}{P \cdot Q}\right).$$

(8a and 8b) show how the difference M between GOS from the national accounts and the sum of payments to observed factors reflects mark-ups and returns to scale (captured by ε / μ) and the influence of unobserved capital inputs (captured by ϕD). The expressions in (8) will be instrumental for the discussion in the following sections.

4. Technical change

In an environment of constant returns to scale, Hicks-neutral technical change can be defined either as a shift of the production function over time (output-based measure) or as a shift of the cost function over time (input-based measure). As producer behaviour has been described by way of a cost function, we shall also use the input-based approach to derive measures of technical change. One important advantage of the cost-based measure is that no assumptions about profit or revenue maximisation have to be made.

If there were an assumption of constant returns to scale, and competitive markets, the choice of the input-based productivity measure would simply be a matter of convenience, with no consequences for results. However, for the moment we have imposed no such a-priori condition, and the input-based measure will in general be different from the output-based measure, as will be shown in section 5.2.(e).

Technical change is measured as a downward shift over time of the total cost function. To derive an analytical expression, TC is differentiated totally and technical change is then given by the negative of the partial derivative of the cost function with respect to the time variable:

$$\text{(9)} \qquad \begin{aligned} -\frac{\partial TC}{\partial t}\frac{1}{TC} = \frac{\partial \ln TC}{\partial t} = \sum_{i=1}^{M} \frac{\partial \ln TC}{\partial \ln Q_i}\frac{d \ln Q_i}{dt} \\ -\left(\frac{d \ln TC}{dt} - \frac{\partial \ln TC}{\partial \ln w}\frac{d \ln w}{dt} - \sum_{i=1}^{N}\frac{\partial \ln TC}{\partial \ln u_i}\frac{d \ln u_i}{dt} - \frac{\partial \ln TC}{\partial \ln \phi}\frac{d \ln \phi}{dt}\right) \end{aligned}$$

To interpret (9), consider its parts in turn. First, there is a Divisia-type output index $\sum_{i=1}^{N} \frac{\partial \ln TC}{\partial \ln Q_i}\frac{d \ln Q_i}{dt}$ that aggregates the growth rates of the quantities of individual outputs. To find a computable expression for the growth rate of output, use (5) and (7) to obtain:

$$\sum_{i=1}^{M}\frac{\partial\ln TC}{\partial\ln Q_i}\frac{d\ln Q_i}{dt}=\sum_{i=1}^{M}\frac{\partial TC}{\partial Q_i}\frac{Q_i}{TC}\frac{d\ln Q_i}{dt}=\sum_{i=1}^{M}P_i\mu_i\frac{Q_i}{TC}\frac{d\ln Q_i}{dt}$$

(10a)
$$=\sum_{i=1}^{M}\frac{P_iQ_i}{P\cdot Q}\mu_i\frac{P\cdot Q}{TC}\frac{d\ln Q_i}{dt}=\frac{P\cdot Q}{TC}\sum_{i=1}^{M}\frac{P_iQ_i}{P\cdot Q}\mu_i\frac{d\ln Q_i}{dt}$$
$$=\varepsilon\sum_{i=1}^{M}\frac{P_iQ_i\mu_i}{P\cdot Q\mu}\frac{d\ln Q_i}{dt}$$

Thus, the correct output aggregate resembles a traditional output aggregate with revenue shares as weights although the latter are corrected for the relative mark-up μ_i/μ and for the scale factor ε.

The second part of the right hand side of (9) measures the difference in the growth rate of total costs and in the rates of the various input prices. In fact, $\left(\frac{\partial\ln TC}{\partial\ln w}\frac{d\ln w}{dt}+\sum_{i=1}^{N}\frac{\partial\ln TC}{\partial\ln u_i}\frac{d\ln u_i}{dt}+\frac{\partial\ln TC}{\partial\ln\phi}\frac{d\ln\phi}{dt}\right)$ is a Divisia index of input prices. This is apparent by invoking the optimality conditions for factor inputs (4a-4c) and inserting them into the above expression which becomes $\left(\frac{wL}{TC}\frac{d\ln w}{dt}+\sum_{i=1}^{N}\frac{u_iK_i}{TC}\frac{d\ln u_i}{dt}+\frac{\phi D}{TC}\frac{d\ln\phi}{dt}\right)$. But the difference between the Divisia index of total costs and the Divisia index of input prices is the Divisia index of input quantities. Consequently, the term in brackets on the right hand side of (9) can be written as

(10b)
$$\left(\frac{d\ln TC}{dt}-\frac{\partial\ln TC}{\partial w}\frac{d\ln w}{dt}-\sum_{i=1}^{N}\frac{\partial\ln TC}{\partial\ln u_i}\frac{d\ln u_i}{dt}-\frac{\partial\ln TC}{\partial\ln\phi}\frac{d\ln\phi}{dt}\right)$$
$$=\left(\frac{wL}{TC}\frac{d\ln L}{dt}+\sum_{i=1}^{N}\frac{u_iK_i}{TC}\frac{d\ln K_i}{dt}+\frac{\phi D}{TC}\frac{d\ln D}{dt}\right)$$

Hence, the theoretical index (9) becomes:

(11a)
$$-\frac{\partial\ln TC}{\partial t}=\varepsilon\sum_{i=1}^{M}\left(\frac{P_iQ_i}{P\cdot Q}\frac{\mu_i}{\mu}\right)\frac{d\ln Q_i}{dt}-\left(\frac{wL}{TC}\frac{d\ln L}{dt}+\sum_{i=1}^{N}\frac{u_iK_i}{TC}\frac{d\ln K_i}{dt}+\frac{\phi D}{TC}\frac{d\ln D}{dt}\right)$$

Turned around, the 'growth accounting' form of (11a) is:

(11b)
$$\varepsilon\sum_{i=1}^{M}\left(\frac{P_iQ_i}{P\cdot Q}\frac{\mu_i}{\mu}\right)\frac{d\ln Q_i}{dt}=\left(\frac{wL}{TC}\frac{d\ln L}{dt}+\sum_{i=1}^{N}\frac{u_iK_i}{TC}\frac{d\ln K_i}{dt}+\frac{\phi D}{TC}\frac{d\ln D}{dt}\right)-\frac{\partial\ln TC}{\partial t}.$$

(11b) delivers an explicit expression for the change in aggregate inputs and outputs. If there were no unobserved factor D, and if mark-up factors and the local scale elasticity were known, (11b) could readily be implemented. In the presence of these unknown parameters, things are more complicated. We start with a proposal for a computable MFP measure and follow with a discussion of its interpretation.

5. Deriving computable measures

There are essentially three strategies for the implementation of (11): (i) to introduce additional, and typically restrictive hypotheses about the size or nature of the unknown variables until an expression has been derived that is both computable and that offers a (seemingly) clear interpretation of productivity growth; (ii) to stay away from invoking additional hypotheses, and define a computable measure of productivity growth but allow for the fact that it may reflect more than pure technology shifts; (iii) to use econometric methods to estimate the unknown parameters and use these parameters to construct the conceptually correct aggregates of outputs, inputs and productivity.

We discard the third possibility simply because it is not a practical way for statistical offices when they have to compute and publish periodic and easily reproducible statistical series. The parametric option is an important one for more research-oriented, one-off projects. As such it may also deliver useful insights into the empirical importance of some of unknown parameters such as the deviations from constant returns to scale or the size of mark-ups. Thus, to assess some of the choices between non-parametric methods as described below, econometric studies (such as Paquet and Robidoux 2001 or Oliveira-Martins et al 1996) can be a very useful tool.

5.1. Apparent multi-factor productivity

We first follow avenue (ii) and propose a measure of 'apparent multi-factor productivity'. Then, additional hypotheses will be used to discuss strategy (i).

For the purpose at hand, let there be an aggregator X that combines quantities of *observable* inputs K and L. Specifically, define

(12) $$\frac{d\ln X}{dt} \equiv \sum_{i=1}^{N} \frac{u_i K_i}{C} \frac{d\ln K_i}{dt} + \frac{wL}{C} \frac{d\ln L}{dt}$$

as a Divisia quantity index of observable inputs, noting that weights correspond to the shares of each input in total *observable* inputs, as $C \equiv wL + u \cdot K$. Next, define the rate of apparent multi-factor productivity growth (AMFP) as the difference between a Divisia quantity index of output and the quantity index of observable inputs as laid out in (12):

(13a) $$AMFP \equiv \frac{d\ln Q}{dt} - \frac{d\ln X}{dt}.$$

The Divisia output index in (13a) is a 'traditional' one, i.e., an average of rates of change of individual outputs, each weighted with its revenue share: $\frac{d\ln Q}{dt} \equiv \sum_{i=1}^{M} \left(\frac{P_i Q_i}{P \cdot Q}\right) \frac{d\ln Q_i}{dt}$ and consequently different from the more general output aggregate identified in (11). The growth accounting equation that corresponds to (13a) is:

(13b) $$\frac{d\ln Q}{dt} = \sum_{i=1}^{N} \frac{u_i K_i}{C} \frac{d\ln K_i}{dt} + \frac{wL}{C} \frac{d\ln L}{dt} + AMFP$$

where, in conjunction with (11), it can be shown that:

$$\text{(13c)} \qquad AMFP = -\frac{\partial \ln TC}{\partial t} + \sum_{i=1}^{M} \frac{P_i Q_i}{P \cdot Q} \left(1 - \frac{\varepsilon \mu_i}{\mu}\right) \frac{d \ln Q_i}{dt} + \frac{\phi D}{TC} \left(\frac{d \ln D}{dt} - \frac{d \ln X}{dt}\right)$$

According to (13b), the direct growth contribution of observed capital inputs and labour is given by the rate of the change of these variables, weighted by their average share in observed costs C. The productivity term AMFP reflects three factors shown in (13c): pure technical change or the shift of the cost function, a term that captures effects of mark-ups and non-constant returns, and a term that captures the effects of the non-observed variable D. Consider the following special cases:

- If there is no unobserved input (D=0), the third term in AMFP disappears and AMFP captures technical change plus a term that reflects the non-constant returns and mark-ups – a result similar to the one developed by Denny, Fuss and Waverman (1981). AMFP will exactly correspond to technical change if there are constant returns to scale ($\varepsilon = 1$) and if the same mark-up factor applies throughout the economy ($\mu_i = \mu$).

- If the volume change of the unobserved input equals the volume change of observed inputs, the third term disappears also and AMFP reflects only technical change and the effects of non-constant returns and mark-ups.

We conclude that – whatever the exact nature of the unobserved factor D – the AMFP computation will capture 'pure' technical change, the growth contributions of unobserved assets and scale effects and the distribution of mark-ups. With the exception of the mark-ups which can be a consequence of market power only (although not every occurrence of mark-ups is necessarily indicative of market power), these effects are technology-related and could be considered analytically meaningful expressions of productivity growth. These effects are not path independent – they vary with the levels and growth rates of observed and non-observed inputs, and the latter depend in turn on prices of inputs and outputs as well as on the size of mark-ups.

The contribution of productivity change to output growth is given by AMFP. Clearly, the interpretation of AMFP has to be kept in mind: it reflects the combined effects of technical change, of non-observed inputs, of non-constant returns to scale and of deviations from perfect competition in product markets. In other words, AMFP is a true 'residual' or a non-theoretic productivity measure. But for many practical purposes, it will fulfil its role as a multi-faceted measure of productivity growth[5]. We note in passing that AMFP could also serve as a useful measure of productivity growth when technical change is of a more general nature, and not necessarily Hicks-neutral.

If one wants to extend the analysis, an additional analytical step could be taken to

5 As can be seen from the list of assets in our empirical implementation, one important asset that is left out is land which is not considered a produced asset by the national accounts. This asset does not grow much so the last term in (13c) is likely to be negative in OECD countries, pointing to a downward bias of AMFP over technical change.

decompose AMFP into its technical change component and into other effects. However, this requires invoking parametric methods of estimation if one does not want to impose competitive behaviour on product markets.

5.2. Invoking additional assumptions

This section follows the approach (i) outlined earlier: additional hypotheses are invoked to deal with the possible presence of unobserved inputs, non-constant returns to scale and mark-ups. Each set of hypotheses is designed so as to lead to a 'correct' measure of MFP in the sense that it reflects Hicks neutral technical change *if the hypotheses hold*. In addition, it is discussed how under these circumstances, the pragmatic AMFP measure would fare: would its use imply an upward or downward bias, or would it coincide with the MFP result?

a) Defining away the unobserved input and assuming common mark-up factors and CRS

If one assumes that there are no unobserved inputs (D=0), and a common mark-up factor in the different output markets ($\mu_i = \mu$), the only possibility to explain a difference between total costs of observed measures and GOS is by way of the combined effects of a positive mark-up and non-constant returns. In this case, the mark-up/returns to scale ratio is given by $\frac{\varepsilon}{\mu} = \frac{M}{P \cdot Q} + 1$, and is determined by the ratio $M / P \cdot Q$, where M corresponds to the difference between non-labour income (GOS) and the sum of observed capital costs and $P \cdot Q$ is the sum of revenues. If empirical information exists on the average mark-up factor μ it can be used to determine ε. Alternatively, information may exist on the average degree of returns to scale in the economy. If not, an additional assumption has to be invoked – typically that of constant returns to scale ($\varepsilon = 1$). Having defined away D, one finds that total costs equal observed costs or TC=C.

In this case, the growth accounting equation (11b) becomes

(14a) $$\sum_{i=1}^{M} \left(\frac{P_i Q_i}{P \cdot Q} \right) \frac{d \ln Q_i}{dt} - \left(\frac{wL}{C} \frac{d \ln L}{dt} + \sum_{i=1}^{N} \frac{u_i K_i}{C} \frac{d \ln K_i}{dt} \right) = MFP1$$

(14b) with $MFP1 = -\frac{\partial \ln TC1}{\partial t}$.

Here, TC1 is the modified cost function that applies under the conditions D=0, $\mu_i = \mu$ and $\varepsilon = 1$. (14b) indicates that MFP1 correctly traces technical change, provided the assumptions D=0, $\mu_i = \mu$ and $\varepsilon = 1$ are accurate. It is easy to see that $MFP1 = \frac{d \ln Q}{dt} - \frac{d \ln X}{dt}$. Thus, if the assumption above holds, the true productivity measure MFP1 coincides with the result obtained from applying an AMFP measure.

b) Assuming proportionality of D and K, absence of mark-up factors and CRS

A second possibility is to allow for an unobserved factor (D>0) but impose marginal cost pricing ($\mu_i = 1$; i=1,…M) and constant returns to scale, i.e., fully competitive output markets. It follows that the entire difference between GOS and the sum of observed payments to capital is identified with payments to the unobserved input: $M = \phi D$. To measure (11), an additional assumption is needed, for example that the rate of change of the unobserved input D equals that of observed capital inputs: $\frac{d\ln D}{dt} = \frac{d\ln K}{dt}$ where $\frac{d\ln K}{dt} \equiv \sum_{i=1}^{N} \frac{u_i K_i}{u \cdot K} \frac{d\ln K_i}{dt}$ is a Divisia quantity index of observed fixed assets. Under these conditions, the growth accounting equation (11b) becomes

(15a)
$$\sum_{i=1}^{M} \frac{P_i Q_i}{P \cdot Q} \frac{d\ln Q_i}{dt} = \frac{wL}{P \cdot Q} \frac{d\ln L}{dt} + \frac{u \cdot K}{P \cdot Q} \frac{d\ln K}{dt} + \frac{M}{P \cdot Q} \frac{d\ln D}{dt} + MFP2$$
$$= \frac{wL}{P \cdot Q} \frac{d\ln L}{dt} + \left(\frac{u \cdot K}{P \cdot Q} + \frac{M}{P \cdot Q} \right) \frac{d\ln K}{dt} + MFP2$$

(15b) with $MFP2 = -\frac{\partial \ln TC}{\partial t}$.

By construction, MFP2 again traces the shift of the cost function TC correctly, as long as the additional hypotheses hold. The measured growth contribution of the observed capital inputs merits further discussion. It is easily verified that $\left(\frac{wL}{P \cdot Q} + \frac{u \cdot K}{P \cdot Q} + \frac{M}{P \cdot Q} \right) = 1$ so that the weight that now attaches to the observed capital inputs, $\left(\frac{u \cdot K}{P \cdot Q} + \frac{M}{P \cdot Q} \right) = \frac{GOS}{P \cdot Q}$, equals the share of GOS in total production which in turn is the complement to the labour share in total income. Thus, the income of the unobserved factor D has been distributed across the observed capital inputs and (15a) can be rewritten as:

(15c) $$\frac{d\ln Q}{dt} = \left(\frac{wL}{P \cdot Q} \frac{d\ln L}{dt} + \frac{GOS}{P \cdot Q} \frac{d\ln K}{dt} \right) + MFP2.$$

(15c) bears a strong resemblance to a model with endogenous net rates of return as described below under case (c). In both cases, the overall rate of growth of capital services, $d\ln K/dt$, enters with the same weight – one minus the labour share in total income. Of course, in the endogenous model, the growth rate of observed capital services $d\ln K^* / dt$ will in general be different from $d\ln K / dt$ in the present case, because each asset's user cost term is based on an endogenous rather than an exogenous rate of return. Nonetheless, as will be apparent from the empirical section, the two MFP measures trace each rather quite closely – at least in the four countries and for the time period under consideration.

Suppose the above assumptions are true but an AMFP measure is applied, what would be the resulting bias with regard to the 'true' MFP2 measure? After some manipulations, it can be shown that

(16) $$\mathrm{AMFP} = \mathrm{MFP2} + \frac{M}{PQ}\left(\frac{d\ln K}{dt} - \frac{d\ln X}{dt}\right).$$

Thus, AMFP will overstate MFP2 if the growth rate of observed capital assets – and by assumption the growth rate of the unobserved asset – exceeds the growth rate of all observed inputs. In the empirical examples presented in section 6, this is the case and AMFP turns out to be consistently higher than MFP2.

c) *Defining away mark-ups and unobserved inputs and assuming CRS*

These are the assumptions invoked when MFP computations rely on endogenous rates of return: output markets are taken as competitive ($\mu_i = 1$; i=1,...M), there are no unobserved factors (D=0) and there are constant returns to scale. The endogenous approach goes back to Christensen and Jorgenson (1969), and has been applied in many studies of productivity growth since, including by statistical offices (BLS 2003). It is the most widely-used methodology but also the one that requires the most restrictive set of assumptions necessary to justify the use of endogenous rates of return[6]. In addition to the above assumptions, there has to be perfect anticipation of asset price changes and depreciation. This implies that $P \cdot Q = wL + u^* \cdot K$. The growth accounting model (11) becomes

(17a) $$\sum_{i=1}^{M}\left(\frac{P_i Q_i}{P \cdot Q}\right)\frac{d\ln Q_i}{dt} = \left(\frac{wL}{P \cdot Q}\frac{d\ln L}{dt} + \sum_{i=1}^{N}\frac{u_i^* K_i}{P \cdot Q}\frac{d\ln K_i}{dt}\right) + \mathrm{MFP3}$$

(17b) where $$\mathrm{MFP3} = -\frac{\partial \ln TC3}{\partial t}.$$

Thus, if the additional restrictions hold, measured productivity change corresponds to the shift of a cost function TC3 with CRS and observed inputs only. As pointed out above, there is similarity with (15c) because (17a) can be re-written as

(18) $$\frac{d\ln Q}{dt} = \left(\frac{wL}{P \cdot Q}\frac{d\ln L}{dt} + \frac{GOS}{P \cdot Q}\frac{d\ln K^*}{dt}\right) + \mathrm{MFP3} \text{ where } \frac{d\ln K^*}{dt} \equiv \sum_{i=1}^{N}\frac{u_i^* K_i}{u^* \cdot K}\frac{d\ln K_i}{dt},$$

so that the contribution of capital assets is the product of the rate of growth of observed capital services and the share of GOS in total output or cost.

If the above assumptions are correct, and if an endogenous rate of return is used, a measurement of productivity with AMFP* would yield the correct result, as

[6] The endogenous rate of return is computed by choosing that net rate of return that just equalizes the sum of user costs of observed assets with non-labour income (GOS for simplicity). Using the same notation for user costs as in footnote 1, this means $GOS = \sum_{i=1}^{N} q^i (r^* + \delta^i - d\ln q^i / dt) K^i$.

$\text{AMFP*} = -\frac{\partial \ln TC3}{\partial t} = \text{MFP3}$ in this case. We have marked AMFP* with an asterisk here to point out that AMFP is based on a capital measure that reflects endogenous rates of return. If AMFP is computed on the basis of exogenous rates, it would clearly differ from MFP3. This is also borne out in the empirical example below. However, no a-priori statement can be made as to the sign of this difference.

d) Defining away mark-ups and the unobserved input but assuming decreasing returns to scale: an input-based measure

This constitutes yet another possibility to deal with the difference between revenues and observed factor payments: the unobserved factor is defined away (D=0) as well as mark-ups of prices over marginal costs ($\mu_i = 1$; i=1,…M), but the production technology is assumed to exhibit decreasing returns to scale ($\varepsilon > 1$). Then, the entire difference between GOS and observed asset rental payments is ascribed to the effects of marginal cost pricing under decreasing returns to scale: $M = (\varepsilon - 1)TC$ and TC=C. Under these circumstances, the returns to scale parameter can be computed: $\varepsilon = M / TC + 1$. Given a value for ε, one obtains from (11) that

(19a) $$\varepsilon \sum_{i=1}^{M} \left(\frac{P_i Q_i}{P \cdot Q} \right) \frac{d \ln Q_i}{dt} = \left(\frac{wL}{TC} \frac{d \ln L}{dt} + \sum_{i=1}^{N} \frac{u_i K_i}{TC} \frac{d \ln K_i}{dt} \right) + \text{MFP4}$$

(19b) where $\text{MFP4} = -\frac{\partial \ln TC4}{\partial t}$

and TC4 is a cost function with decreasing returns to scale and observed factor inputs only. Some more discussion is useful here. First, because all costs are observed, TC=C and (19a) can be written as

(19c) $$\varepsilon \frac{d \ln Q}{dt} = \frac{d \ln X}{dt} + \text{MFP4}.$$

We note in passing that the same growth accounting equation and/or productivity measure MFP4 could have been derived from a model with a constant returns to scale cost function in observed and unobserved inputs, but assuming that the quantity of the unobserved input is positive and fixed[7]. The unobserved input then acts as the additional cost factor that is equivalent to a decreasing returns to scale technology.

If the assumptions above are correct, how does MFP4 relate to AMFP? It is easily established that under these circumstances, $\text{AMFP} = \text{MFP4} - (\varepsilon - 1)\frac{d \ln Q}{dt}$. If returns to scale are decreasing ($\varepsilon > 1$), and if the quantity of output increases ($d \ln Q / dt > 0$), AMFP would turn out to be smaller than MFP4 as AMFP captures both the effects of pure technical change and non-constant returns. This is borne out by the empirical observations in section 6.

[7] The idea is based on Diewert and Nakamura (2003) who introduce an unknown variable into a cost-function to deal with decreasing returns to scale.

e) *Defining away mark-ups and the unobserved input but assuming decreasing returns to scale: an output-based measure*

It is well known that a production technology with non-constant returns to scale gives rise to several productivity measures (see, for example, Balk (1998)). In particular there are differences between output-based measures of technology such as the shift of a production function or of a revenue function over time and input-based measures of technology such as the shift of a cost function or of an input distance function over time. In the sections above, the analysis has been based on a cost function, i.e., an input-based measure. To introduce an alternative and output-based measure of technical change, we shall consider a revenue function and its shift over time. As in section d), we assume that there is no unobserved input and that there are no mark-ups. As a consequence, the value of M is entirely associated with the decreasing returns to scale ($M = (\varepsilon - 1)TC$) and total costs equal observed costs: TC=C.

To derive the output-based productivity measure, consider the revenue function[8] R, defined so as to show maximum revenues given a vector of inputs and given a vector of output prices:

(20) $$R(P, L, K, t) = \max_{Q} \{P \cdot Q : (Q, L, K) \text{ belongs to } Z(t)\}$$

Diewert (1983) first used a revenue function to define a theoretical productivity index, albeit in discrete time. We follow his approach and define the continuous-time equivalent as the partial derivative of the revenue function with respect to time: total differentiation of R yields the following output-based measure of technical change:

(21) $$\frac{\partial \ln R}{\partial t} = \frac{d \ln R}{dt} - \sum_{i=1}^{M} \frac{\partial \ln R}{\partial \ln P_i} \frac{d \ln P_i}{dt} - \frac{\partial \ln R}{\partial \ln L} \frac{d \ln L}{dt} - \sum_{i=1}^{N} \frac{\partial \ln R}{\partial \ln K_i} \frac{d \ln K_i}{dt}.$$

To derive a computable measure of the output-based productivity measure, an additional assumption has to be introduced: revenue-maximising behaviour on the part of producers. Then, observed revenues equal maximum revenues: $P \cdot Q = R$. If in addition, firms are price takers, one gets $Q_i = \partial R / \partial P_i$. It is then straightforward to obtain a computable expression for the elasticity of revenues with respect to output prices: $\frac{\partial \ln R}{\partial \ln P_i} = \frac{P_i Q_i}{R} = \frac{P_i Q_i}{P \cdot Q}$. It should be noted that the assumption of revenue maximisation and price taking on output markets were not necessary for the derivation of the input-based measure. Thus, the output-based productivity statistics requires more stringent assumptions than the input-based statistic.

Next, define the Divisia decomposition of total revenues into a price and a quantity index as:

[8] The concept of a revenue function is due to Samuelson (1953-54).

(22) $$\frac{d\ln R}{dt} = \sum_{i=1}^{M} \frac{P_i Q_i}{P \cdot Q} \frac{d\ln P_i}{dt} + \sum_{i=1}^{M} \frac{P_i Q_i}{P \cdot Q} \frac{d\ln Q_i}{dt}.$$

The first two expressions on the right hand side of (21) are then equivalent to a Divisia quantity index of outputs: $\frac{d\ln R}{dt} - \sum_{i=1}^{M} \frac{P_i Q_i}{P \cdot Q} \frac{d\ln P_i}{dt} = \sum_{i=1}^{M} \frac{P_i Q_i}{P \cdot Q} \frac{d\ln Q_i}{dt} \equiv \frac{d\ln Q}{dt}$.

To find computable expressions for the input elasticities of the revenue function, we invoke profit-maximising behaviour of producers. This implies that they solve a maximisation problem of the kind $\max_{L,K}\{R(P,L,K) - wL - u \cdot K\}$. The first order conditions for a maximum are $\partial R / \partial L = w$ and $\partial R / \partial K_i = u_i$ (i=1,...N). Consequently, $\partial \ln R / \partial \ln L = wL / R$ and $\partial \ln R / \partial \ln K_i = u_i K_i / R$ (i=1,...N). Then, the third and fourth expression on the right-hand side of (21) read as

(23) $$\begin{aligned} \frac{\partial \ln R}{\partial \ln L}\frac{d\ln L}{dt} + \sum_{i=1}^{N} \frac{\partial \ln R}{\partial \ln K_i}\frac{d\ln K_i}{dt} &= \frac{wL}{R}\frac{d\ln L}{dt} + \sum_{i=1}^{N} \frac{u_i K_i}{R}\frac{d\ln K_i}{dt} \\ &= \frac{C}{R}\left(\frac{wL}{C}\frac{d\ln L}{dt} + \sum_{i=1}^{N} \frac{u_i K_i}{C}\frac{d\ln K_i}{dt}\right). \\ &= \frac{C}{R}\frac{d\ln X}{dt} \end{aligned}$$

But $C / R = TC / P \cdot C = 1/\varepsilon$ and the final, computable, expression for the output side-based productivity measure in (21) is

(24) $$\frac{\partial \ln R}{\partial t} = \frac{d\ln Q}{dt} - \frac{1}{\varepsilon}\frac{d\ln X}{dt} = MFP5.$$

The link to AMFP is readily established: one finds that

(25) $$MFP5 = \frac{d\ln Q}{dt} - \frac{d\ln X}{dt} - \left(\frac{1}{\varepsilon} - 1\right)\frac{d\ln X}{dt} = AMFP + \left(1 - \frac{1}{\varepsilon}\right)\frac{d\ln X}{dt}.$$

Thus, MFP5 will exceed AMFP if the quantity index of inputs grows at a positive rate and this can be observed in the country examples in section 6.

f) A note on increasing returns to scale

There is no reason to believe that returns to scale may not be locally increasing and so this case has to be treated as well. Suppose, therefore that $\varepsilon < 1$. Unless one admits a situation where M<0 – implying losses for producers – increasing returns to scale have to come in tandem with positive mark-ups over marginal costs. Thus, to have M>0 under IRS, μ_i has to be positive but less than unity for at least one product.

Then, $M = P \cdot Q(1 - \mu / \varepsilon)$ if we assume that there is no unobserved input (D=0). Under these assumptions, the growth accounting and productivity equation (11) takes the following form:

(26) $$\frac{\varepsilon}{\mu}\sum_{i=1}^{M}\frac{P_i Q_i}{P\cdot Q}\mu_i\frac{d\ln Q_i}{dt}=\frac{d\ln X}{dt}+MFP6\,.$$

While (26) is a valid measure for the shift in cost function TC6 with increasing returns to scale and without unobserved inputs, it is apparent that with the observable information on prices, quantities and factor remuneration (26) cannot be computed. Although ε/μ is known, there is not enough information to deduce values for product-specific mark up factors μ_i. Extraneous information is required, otherwise MFP6 cannot be computed. While such information may be available, this will rarely be the case on a timely and comprehensive basis. For example, Oliveira-Martins et al. (1996) estimated mark-up ratios for 14 OECD countries by industry and typically found positive mark-ups. But one-off studies are quickly outdated and information about mark-ups is not easily applied to other, far-off periods. Also, industry-level mark-up estimates are frequently confined to manufacturing industries, leaving open the important branches in the service sector. Overall, it would not seem practical for a statistical office to rely on mark-up estimates for purposes of productivity statistics. For the same reason, we were not in a position to compute empirical results for MFP6.

6. Empirical implementation

After the theoretical derivations in section 5, we shall now move on to empirical considerations. Several questions arise: one concerns index numbers – how should the continuous-time formulae be translated into discrete index number formulae to accommodate the fact that data comes in discrete form? A second question relates to how exactly some of the variables should be measured, in particular capital services and user costs of capital. Finally, we wish to compare the various productivity measures to get a sense for the importance of the choice of assumptions.

6.1. Choice of index number formulae

Concerning the index number issue, our approach has been one of approximating the continuous-time divisia indices in the theoretical part of the paper by Törnqvist-type indices for the present empirical part. We are aware of the methodological shortcomings of this procedure: this discrete approximation is essentially an arbitrary choice[9], not rigorously backed up by theory. A more thorough procedure would have been to start out with discrete formulations for the cost and revenue functions and then derive the appropriate index number formulae together with the productivity measure. However, we feel that the theoretical advantages of a full derivation in discrete time are outweighed by the algebraic complications that such an approach brings along: in particular, there is

[9] That nearly all common index number formulae can be considered valid discrete approximations to the Divisia index has already been shown by Frisch (1936). For a more recent statement, see Diewert (1980) or Balk (2000).

a host of interaction terms which add little to the message delivered in the present paper but which would have rendered the exposition much less readable.

For the purpose at hand then, we chose the following Törnqvist-type approximations to the above Divisia-type formulations of the various productivity indices:

(27a) $$\frac{d\ln Q}{dt} \approx \sum_{i=1}^{M} \tfrac{1}{2}\left(\frac{P_i^t Q_i^t}{P^t \cdot Q^t} + \frac{P_i^{t-1} Q_i^{t-1}}{P^{t-1} \cdot Q^{t-1}}\right) \ln\left(\frac{Q_i^t}{Q_i^{t-1}}\right) \equiv \ln\left(\frac{Q^t}{Q^{t-1}}\right)$$

(27b) $$\frac{d\ln X}{dt} \approx \tfrac{1}{2}\left(\frac{w^t L^t}{C^t} + \frac{w^{t-1} L^{t-1}}{C^{t-1}}\right) \ln\left(\frac{L^t}{L^{t-1}}\right) + \sum_{i=1}^{N} \tfrac{1}{2}\left(\frac{u_i^t K_i^t}{C^t} + \frac{u_i^{t-1} K_i^{t-1}}{C^{t-1}}\right) \ln\left(\frac{K_i^t}{K_i^{t-1}}\right) \equiv \ln\left(\frac{X^t}{X^{t-1}}\right)$$

(27c) $$AMFP^{t/t-1} = \ln\left(\frac{Q^t}{Q^{t-1}}\right) - \ln\left(\frac{X^t}{X^{t-1}}\right)$$

(27b) $$MFP1^{t/t-1} = AMFP^{t/t-1}$$

(27c) $$MFP2^{t/t-1} = \ln\left(\frac{Q^t}{Q^{t-1}}\right) - \tfrac{1}{2}\left(\frac{w^t L^t}{P^t \cdot Q^t} + \frac{w^{t-1} L^{t-1}}{P^{t-1} \cdot Q^{t-1}}\right) \ln\left(\frac{L^t}{L^{t-1}}\right) - \sum_{i=1}^{N} \tfrac{1}{2}\left(\frac{u_i^t K_i^t + M^t}{P^t \cdot Q^t} + \frac{u_i^{t-1} K_i^{t-1} + M^{t-1}}{P^{t-1} \cdot Q^{t-1}}\right) \ln\left(\frac{K_i^t}{K_i^{t-1}}\right)$$

(27d) $$MFP3^{t/t-1} = \ln\left(\frac{Q^t}{Q^{t-1}}\right) - \tfrac{1}{2}\left(\frac{w^t L^t}{P^t \cdot Q^t} + \frac{w^{t-1} L^{t-1}}{P^{t-1} \cdot Q^{t-1}}\right) \ln\left(\frac{L^t}{L^{t-1}}\right) - \sum_{i=1}^{N} \tfrac{1}{2}\left(\frac{u_i^{*t} K_i^t}{P^t \cdot Q^t} + \frac{u_i^{*t-1} K_i^{t-1}}{P^{t-1} \cdot Q^{t-1}}\right) \ln\left(\frac{K_i^t}{K_i^{t-1}}\right)$$

(27e) $$MFP4^{t/t-1} = \tfrac{1}{2}\left(\varepsilon^t + \varepsilon^{t-1}\right) \ln\left(\frac{Q^t}{Q^{t-1}}\right) - \ln\left(\frac{X^t}{X^{t-1}}\right)$$

(27f) $$MFP5^{t/t-1} = \ln\left(\frac{Q^t}{Q^{t-1}}\right) - \tfrac{1}{2}\left(\frac{1}{\varepsilon^t} + \frac{1}{\varepsilon^{t-1}}\right) \ln\left(\frac{X^t}{X^{t-1}}\right)$$

6.2. Measuring outputs and inputs

The empirical productivity measures developed in the present paper all relate to the total economy. This reflects data constraints more than a preferred choice which would have been to limit computations to the corporate or business sector. However, neither capital input measures nor hours worked are easily available in such a sectoral breakdown and calculations remain at the aggregate level, in line with the data available from the *OECD Productivity Database*[10].

(a) Outputs

More specifically, value-added has been measured at basic prices, i.e., excluding taxes on products and including subsidies on products, because this valuation constitutes the economically relevant variable from a producer perspective. Time series on value-added and net indirect taxes were taken from the *OECD Annual National Accounts*.

A second adjustment to aggregate value-added is also required to maintain consistency between input and output data: capital input in the *OECD Productivity Database* is limited to non-residential, fixed assets in scope and consequently, the value-added produced with residential assets should be excluded from productivity calculations. Thus, total value-added has to be corrected for the production of owner-occupiers[11].

One notes that both adjustments (valuation of output at basic prices and exclusion of the production of owner-occupiers of dwellings) have immediate consequence for the size of the endogenous rate of return as computed in MFP3 and for the weights that attach to capital and labour in MFP2. AMFP, on the other hand, is influenced by these adjustments only to the extent that they bear on the volume growth rate of output. However, current-price value-added does not enter the AMFP computation because labour and capital weights are determined independent of the output measure. This is a distinct advantage in the presence of incomplete output data[12].

(b) Inputs

Labour input is measured as total hours worked in the economy – a difficult task in particular at the international level. Even so, this remains an imperfect measure: no account is taken of labour quality as hours of persons with skills and experience are simply added up. A more appropriate index of labour input would weight different types of hours worked with their corresponding share in overall compensation. The most important measurement issues are described in a note available on the site of the *OECD Productivity Database*.

Capital inputs are derived on the basis of the perpetual inventory method. The estimation of capital service flows starts with identifying those assets that correspond to

[10] www.oecd.org/statistics/productivity

[11] The need for this exclusion and possible consequences for the measurement of the endogenous rates of return were pointed out to me by Mathilde Mas (University of Valencia).

[12] For example, the available OECD national accounts data do not permit to single out the production of the owner-occupied dwellings industry – only the parent aggregate with real estate, renting and business activities is available. For purposes of the present computations, an assumption had to be made that the production of owner-occupiers accounts for one third of the entire industry. Obviously, this introduces a potential bias in those computations that depend on such an adjustment.

the breakdown currently available from the OECD/Eurostat National Accounts questionnaire, augmented by information on information and communication technology assets. Only non-residential gross fixed capital formation is considered, and in particular, seven types of assets or products:

Type of product/asset
Products of agriculture, metal products and machinery
Of which: IT Hardware Communications equipment Other
Transport equipment
Non-residential construction
Other products Of which: Software Other

Investment. For each type of asset, a time series of current-price investment expenditure and a time series of corresponding price indices is established, starting with the year 1960. For many countries, this involves a certain amount of estimates, in particular for the period 1960-80. Such estimates are typically based on national accounts data prior to the introduction of SNA93, or on relationships between different types of assets that are established for recent periods and projected backwards. For purposes of exposition of the methodology, call current price investment series for asset type *i* in year t IN_t^i (i=1,2,..7) and the corresponding price index q_t^i. Price indices are normalised to the reference year 1995 where $q_t^i = 1$.

Price indices should be constant quality deflators that reflect price changes for a given investment good. This is particularly important for those items that have seen rapid quality change, in particular information and communication technology assets. There, observed price changes of 'computer boxes' have to be quality-adjusted for comparison of different vintages. Wyckoff (1995) was one of the first to point out that the large differences that could be observed between computer price indices in OECD countries were likely much more a reflection of differences in statistical methodology than true differences in price changes. In particular, those countries that employ hedonic methods to construct ICT deflators tend to register a larger drop in ICT prices than countries that do not. Schreyer (2000) used a set of 'harmonised' deflators to control for some of the differences in methodology. We follow this approach and assume that the ratios between ICT and non-ICT asset prices evolve in a similar manner across countries, using the United States as the benchmark. Although no claim is made that the 'harmonised' deflator is necessarily the correct price index for a given country, the possible error due to using a harmonised price index is smaller than the bias arising from comparing capital services based on national deflators[13].

[13] See Schreyer et al. (2003) for details. There is a difficulty with the harmonised deflator that should be noted. From an accounting perspective, adjusting the price index for investment goods for any country implies an adjustment of the volume index of output. In most cases, such an adjustment would increase the measured

Productive stocks. Given price and volume series for investment goods, for each of the (supposedly) homogenous asset types, a productive stock S^i_t has been constructed as follows:

(28) $S^i_t = \sum_{\tau=1}^{T^i} \left(IN^i_{t-\tau} / q^i_{t-\tau,0}\right) h^i_\tau F^i_\tau$, i=1,...7.

In this expression, the productive stock of asset i at the beginning of period t is the sum over all past investments in this asset, where current price investment in past periods, $IN^i_{t-\tau}$ has been deflated with the purchase price index of new capital goods, $q^i_{t-\tau,0}$. T^i represents the maximum service life of asset type i.

Because past vintages of capital goods are less efficient than new ones, an age efficiency function h^i_τ has been applied. It describes the efficiency time profile of an asset, conditional on its survival and is defined as a hyperbolic function of the form used by the United States Bureau of Labor Statistics (BLS 1983) $h^i_\tau = \left(T^i - \tau\right)/\left(T^i - \beta\tau\right)$.

Capital goods of the same type purchased in the same year do not generally retire at the same moment. More likely, there is a retirement distribution around a mean service life. In the present calculations, a normal distribution with a standard deviation of 25% of the average service life is chosen to represent probability of retirement. The distribution was truncated at an assumed maximum service life of 1.5 time the average service life. The parameter F^i_τ is the cumulative value of this distribution, describing the probability of survival over the cohort's life span. The following average service lives are assumed for the different assets: 7 years for IT equipment, 15 years for communications equipment, other equipment and transport equipment, 60 years for non-residential structures, 3 years for software and 7 years for remaining other products. The parameter β in the age-efficiency function was set to 0.8. Service lives and parameter values follow BLS practice.

User costs of capital. In a fully functioning asset market, the purchase price of an asset will equal the discounted flow of the value of services that the asset is expected to generate in the future. This equilibrium condition is used to derive the rental price or user cost expression for assets. Let $q^i_{t,0}$ denote the purchase price in year t of a new (zero-year old) asset of type i, and let $u^i_{t+\tau,\tau}$ be the rental price that this asset is expected to fetch in period $t+\tau$ (first subscript to the right) when the asset will be of age τ (second subscript to the right). With r as the nominal discount rate valid at time t, the asset market equilibrium condition for a new asset (age zero) becomes:

(29) $q^i_{t,0} = \sum_{\tau=0}^{\infty} u^i_{t+\tau+1,\tau} (1+r)^{-(\tau+1)}$

This formulation implies that rentals are paid at the end of each period. To solve this expression for the rental price, the price for a one year old asset in the period t+1 is

rate of volume output change. At the same time, effects on the economy-wide rate of GDP growth appear to be relatively small (see Schreyer (2002) for a discussion).

computed as $q^i_{t+1,1} = \sum_{\tau=0}^{\infty} u^i_{t+\tau+2,\tau+1}(1+r)^{-(\tau+1)}$ and then subtracted from the expression above to obtain $u^i_{t+1,0} = q^i_{t,0}(1+r) - q^i_{t+1,1}$ or $u^i_{t,0} = q^i_{t-1,0}(1+r) - q^i_{t,1}$ which can be transformed into

(30) $$u^i_{t,0} = q^i_{t-1,0}\left(r + d^i_{t,0} - \zeta^i_t + \delta^i \zeta^i_t\right).$$

This is the user cost formulation[14] applied in the present paper, where the rate of depreciation of asset i has been defined as $d^i_{t,s} \equiv 1 - q^i_{t,s+1} / q^i_{t,s}$ and the rate of price change of the same asset as $\zeta^i_t \equiv q^i_{t,s} / q^i_{t-1,s} - 1$. One notes that the different variables in the user cost equation are expected variables because they invoke knowledge about price changes in future periods.

These expectations govern the rental price. The System of National Accounts, to which capital stock data should tie into, is based on ex-post prices, observed in the context of actual transactions. Would the use of user cost expressions such as the one above then be in contradiction with the principles of national accounts?

In our view, the answer is 'no'. Note that the presence of expectations does not make the user cost term less 'real': transactions are concluded at this price, even if with hindsight (ex post) the expectations underlying it may turn out to be wrong. This is most apparent when one thinks of a case where capital goods are actually rented: the observed rental price characterises the transaction and is the relevant market price, typically dependent on expectations on the side of the lessor and the lessee. Nobody would challenge using such observed prices in the national accounts. If rental prices are not observable, values have to be imputed, and the expression above indicates how this can be done on the basis of economic theory. Imputations are numerous in the national accounts, and in this sense, the imputation of user costs would not constitute an exception.

Thus, it is not the presence of expected variable as such that is at issue. The real question from a capital and productivity measurement viewpoint is: is the realised but unobserved marginal productivity of fixed assets better approximated by an ex-ante or by an ex-post measure of user costs?

The distinction between expected or ex-ante user costs has been discussed by Berndt and Fuss (1986) Harper et al. (1989), Diewert (2001), Berndt (1990) in his discussion of Hulten (1990) and Hill and Hill (2003). Earlier, it was concluded that the importance of the distinction between ex-ante and ex-post measures lies in their capacity to approximate the realised marginal productivity of capital assets. On this matter, Berndt (1990) points out that: "…if one wants to use a measure of capital to calculate actual multifactor productivity growth, then theory tells us quite clearly that we should weight the various traditionally measured capital inputs by their realised marginal products, not their expected marginal products. This means that in choosing capital service price

[14] Jorgenson and Yun (2001) show how tax considerations enter the user cost of capital and how they affect measured economic performance. While one of the projects of expansion of the OECD Productivity Database, such fiscal parameters are at present not considered in our set of user costs and capital measures.

weights, on should employ shadow values or ex post rates of return, and not the ex ante rates of return that are appropriate in the investment context."

While we concur with Berndt's statement that for purposes of productivity measurement, realised marginal products are the appropriate weights, we wish to point out that this does not necessarily imply that ex post rates of return are always the preferred approximation to realised marginal productivity. Suppose that a capital asset is rented by a producer at a given, pre-agreed rental price to be paid by the end of the period. Independent of the ex-post rental price will the lessee of the asset use it in his production process as planned. Then, the marginal productivity of the asset in the production process would best be approximated by the ex-ante rental price which is the price at which the rental transaction took place.

Take another case of an owner/producer and suppose that there has been investment at the beginning of the period in line with the ex-ante user cost. Now let there be a change in market conditions that lead to a modification of expectations and of user costs. If capital is fully flexible and can be adjusted continuously, it will be done so in line with the new user cost term. But the user cost term remains one governed by expectations, even though expectations may have changed. Only when capital cannot be adjusted, the ex-post user cost term would furnish the preferred approximation to the realised marginal productivity of an asset. This is the case that Berndt (1990) and Berndt and Fuss (1986) have in mind and it relies on quasi-fixity of capital in the production process. In other words, there is no general conclusion that ex-post user cost measures should always be preferred to ex-ante one for purposes of measuring and aggregating capital input.

There is another conceptual difficulty with ex post user costs: the computation of the realised rates of return is commonly done by choosing a rate of return so that the ensuing user cost and total value of capital services just exhausts the measured gross operating surplus available from the national accounts. This computation relies, however, on the assumption that there be only one ex-post rate of return across all assets. While equalisation of rates of return across assets is a natural assumption in an ex-ante context, it is much harder to justify in an ex-post context, and a state of disequilibrium. Thus, we would be imposing an equilibrium condition to implement an (ex-post) measure that was specifically chosen on the grounds that it captures the nature of a situation of disequilibrium.

Diewert (2001) also points out that while the ex-post measure (of the nominal rate of return) is widely used in empirical research, it is subject to measurement error and it may not reflect the economic conditions facing producers at the beginning of the period.

Note a practical argument against the ex-post rate: its calculation requires information on the level of the productive capital stock at current prices (or alternatively on the wealth stock at current prices). But levels of capital stocks tend to be less reliable statistics than their rates of change, in particular when long historical investment series have to be estimated. This problem does not arise when user costs and nominal rates of return are of an ex-ante nature and therefore exogenous variables. On the other hand, ex-post rates of return are of interest as such, and straightforward to compute. In sum then, there is no clear conclusion on this matter. For the present work, however, we gave preference to an ex-ante approach – mainly because it allows us to develop capital service measures independently from measures of labour compensation, gross operating surplus and mixed income in the national accounts.

Exogenous net rate of return. To compute the net rate of return, we follow a suggestion by Diewert (2001) and use as a starting point a constant value for the expected real interest rate rr. The constant real rate is computed by taking a series of annual observed nominal rates (un-weighted average of interest rate with different maturities[15]) and deflating them by the consumer price index. The resulting series of real interest rates is averaged over the period (1980-2000) to yield a constant value for rr. The expected nominal interest rate for every year is then computed as $r_t = (rr+1)(1+p_t)-1$ where p is the expected value of an overall deflator, the consumer price index.

To obtain a measure for p, the expected overall inflation, we construct a 5-year centred moving average of the rate of change of the consumer price index $p_t = \sum_{s=-2}^{+2} CPI_{t-s}$ where CPI_t is the annual percentage change of the consumer price index. This yields the expected rate of overall price change and, by implication, the nominal net rate of return.

Expected asset price changes, another element in the user cost equation, are derived as a smoothed series of actual asset price change: a simple 5-year centred moving average serves as a filter.

Depreciation rates have been computed using the definition given above $d^i_{t,0} \equiv 1 - q^i_{t,1}/q^i_{t,0}$: the rate of depreciation for a new asset equals one minus the ratio of the market price for a one-year old asset over the market price for a new asset. While the market price for a new asset can be observed directly, the vintage price for a one-year old asset has to be computed, using the asset market equilibrium condition (29), the age-efficiency function h and the discount rate.

6.3. Results

Tables 1 -4 summarise empirical results for Canada, France, the United States and Japan. They show the rates of change of output (GDP) and labour input as well as the volume changes in capital services, alternatively based on exogenous and endogenous rates of return as well as the various MFP measures. The first observation is that moving from an endogenous to an exogenous rate of return leads to a rise in the observed measure of capital input – at least in the case of the countries considered and for the period at hand. Also, labour and capital shares turn out to be quite different when based on total costs rather than total revenue.

The second panel in the table reviews results for the five alternative MFP measures presented in the text above. It is immediately apparent that the different options – each

[15] These are the average bank rate, the bank rate on prime loans, long-term government bond yields, short-term government bond yields, the interest rate on a 90 day bank fixed deposit and the treasury bill rate.

associated with a particular set of assumptions about market structures or production technology – can lead to considerable variation in the resulting MFP measures, France being a noticeable exception. Unless one employs econometric techniques or unless a-priori knowledge about technology and market structure are available, it will be difficult to choose between the different options. Also, every different MFP measure implies a different message about the relative contribution of capital services to output growth. Throughout our countries, measured productivity growth turns out to be slowest when based on endogenous rate models (MFP3) or when assuming proportionality between capital input and an unobserved factor (MFP2). Output-based productivity measures under decreasing returns to scale (MFP5) are generally the fastest-growing item in all countries, followed by the input-based productivity measure with decreasing returns to scale (MFP4).

The figures for each country plot again the set of MFP measures over time. It is remarkable that a simple geometric average across the five specific MFP measures yields a time series that is traced very close by the simple AMFP measure. In the absence of a-priori information on mark-ups, returns to scale, or unobserved assets, the choice of a measure that is close to the average of the different options may be a reasonable one. This is also one of our main conclusions.

Canada: basic series							
	Output	Hours worked		Capital services			
				Exogenous RoR		Endogenous RoR	
	Volume index	Cost share	Volume index	Cost share	Volume index	Revenue share	Volume index
1985	100.0	71.70%	100.0	28.30%	100.0	33.30%	100.0
1986	102.4	72.69%	102.9	27.31%	105.0	32.28%	105.8
1987	106.7	73.54%	106.4	26.46%	110.6	32.41%	112.1
1988	112.0	73.69%	110.7	26.31%	116.9	32.07%	119.2
1989	114.9	71.39%	112.9	28.61%	123.1	31.36%	126.5
1990	115.1	70.34%	112.6	29.66%	128.7	30.49%	133.1
1991	112.6	69.61%	109.2	30.39%	133.7	28.71%	139.1
1992	113.6	73.23%	108.0	26.77%	138.3	28.19%	144.5
1993	116.3	72.76%	110.1	27.24%	142.7	28.52%	149.7
1994	121.8	75.35%	113.3	24.65%	147.6	30.91%	155.4
1995	125.2	74.38%	114.6	25.62%	152.9	32.32%	161.2
1996	127.3	76.10%	116.8	23.90%	158.6	32.16%	167.3
1997	132.7	74.31%	118.5	25.69%	166.5	32.65%	175.5
1998	138.1	74.22%	121.5	25.78%	175.3	31.48%	184.6
1999	145.8	73.00%	125.4	27.00%	184.9	32.94%	194.5
2000	153.5	74.31%	128.4	25.69%	194.3	34.56%	204.3
2001	156.4	72.66%	128.4	27.34%	202.9	..	204.3
2002	161.6	72.29%	130.3	27.71%	209.7	..	204.3
85-90	2.81%	72.23%	2.37%	27.77%	5.05%	31.98%	5.71%
90-95	1.70%	72.61%	0.35%	27.39%	3.45%	29.86%	3.84%
95-00	4.07%	74.38%	2.28%	25.62%	4.80%	32.69%	4.73%
95-02	3.64%	73.91%	1.84%	26.09%	4.51%	32.69%	3.38%

Canada: productivity measures						
	MFP1 =AMFP	MFP2	MFP3	MFP4	MFP5	Average
1985	100.0	100.0	100.0	100.0	100.0	100.0
1986	99.0	98.9	98.6	99.2	99.2	99.0
1987	99.2	99.1	98.6	99.8	99.8	99.3
1988	99.7	99.4	98.8	100.6	100.6	99.8
1989	99.3	98.9	98.1	100.4	100.4	99.4
1990	98.4	97.9	96.9	99.5	99.5	98.5
1991	97.3	96.9	95.7	98.4	98.4	97.3
1992	98.0	97.5	96.2	99.1	99.0	98.0
1993	98.1	97.6	96.2	99.2	99.2	98.0
1994	99.7	99.2	97.7	101.1	101.0	99.7
1995	100.7	100.1	98.4	102.4	102.2	100.8
1996	100.0	99.2	97.6	101.9	101.7	100.0
1997	101.9	100.8	99.2	104.3	103.8	102.0
1998	102.7	101.5	99.9	105.5	105.0	102.9
1999	104.5	103.1	101.5	107.9	107.1	104.8
2000	106.7	105.1	103.5	110.7	109.7	107.1
2001	107.5	..	..	110.7	..	109.1
2002	108.8	..	..	108.5	..	108.7
85-90	-0.31%	-0.42%	-0.63%	-0.09%	-0.10%	-0.31%
90-95	0.46%	0.43%	0.32%	0.58%	0.53%	0.46%
95-00	1.15%	0.97%	0.99%	1.56%	1.42%	1.22%
95-02	1.10%	..	..	0.82%	..	1.08%

Source : OECD Productivity Database May 2004.

France: basic series							
	Output	Hours worked		Capital services			
				Exogenous RoR		Endogenous RoR	
	Volume index	Cost share	Volume index	Cost share	Volume index	Revenue share	Volume index
1985	100.0	69.53%	100.0	30.47%	100.0	24.32%	100.0
1986	102.4	70.42%	99.8	29.58%	102.7	26.56%	102.8
1987	105.0	71.36%	100.3	28.64%	105.7	27.46%	105.9
1988	109.8	71.48%	101.3	28.52%	109.4	28.88%	109.6
1989	114.4	70.06%	101.9	29.94%	113.7	30.06%	113.9
1990	117.4	69.81%	102.8	30.19%	118.4	29.80%	118.6
1991	118.6	69.11%	102.4	30.89%	122.7	29.94%	123.0
1992	120.4	68.10%	101.8	31.90%	126.5	30.39%	126.9
1993	119.3	68.26%	99.7	31.74%	129.4	30.59%	129.9
1994	121.8	67.91%	99.5	32.09%	132.3	31.64%	132.7
1995	123.8	68.67%	98.8	31.33%	135.0	31.89%	135.4
1996	125.2	68.86%	99.5	31.14%	137.7	31.71%	138.2
1997	127.5	69.50%	99.5	30.50%	140.5	32.30%	141.0
1998	131.9	70.50%	100.3	29.50%	144.0	33.09%	144.4
1999	136.1	71.89%	101.9	28.11%	148.5	32.87%	148.6
2000	141.3	71.12%	101.3	28.88%	153.6	33.21%	153.5
2001	144.2	68.83%	101.5	31.17%	158.5	33.00%	158.3
2002	146.0	69.40%	101.1	30.60%	161.7	32.38%	161.5
85-90	3.21%	70.44%	0.56%	29.56%	3.37%	27.85%	3.42%
90-95	1.06%	68.64%	-0.79%	31.36%	2.62%	30.71%	2.64%
95-00	2.64%	70.09%	0.50%	29.91%	2.59%	32.51%	2.50%
95-02	2.36%	69.85%	0.33%	30.15%	2.59%	32.56%	2.52%

France: productivity measures						
	MFP1 =AMFP	MFP2	MFP3	MFP4	MFP5	Average
1985	100.0	100.0	100.0	100.0	100.0	100.0
1986	101.8	101.9	101.9	101.6	101.7	101.8
1987	103.1	103.3	103.2	102.9	103.0	103.1
1988	106.0	106.2	106.2	105.7	105.9	106.0
1989	108.8	109.0	108.9	108.5	108.7	108.8
1990	109.6	109.8	109.7	109.3	109.5	109.6
1991	109.8	110.0	109.9	109.5	109.7	109.8
1992	110.8	111.1	111.0	110.5	110.7	110.8
1993	110.5	110.9	110.8	110.3	110.4	110.6
1994	112.2	112.6	112.5	111.9	112.1	112.3
1995	113.9	114.3	114.1	113.6	113.8	113.9
1996	113.9	114.3	114.2	113.6	113.8	114.0
1997	115.3	115.7	115.6	115.0	115.2	115.4
1998	117.7	118.1	118.0	117.6	117.7	117.8
1999	119.1	119.4	119.3	119.2	119.2	119.2
2000	122.9	122.9	123.0	123.3	123.1	123.1
2001	124.2	124.1	124.1	124.7	124.4	124.3
2002	125.2	125.1	125.1	125.8	125.4	125.3
85-90	1.83%	1.86%	1.85%	1.78%	1.81%	1.83%
90-95	0.77%	0.80%	0.80%	0.77%	0.77%	0.78%
95-00	1.53%	1.46%	1.49%	1.65%	1.57%	1.54%
95-02	1.35%	1.29%	1.31%	1.46%	1.39%	1.36%

Source : OECD Productivity Database May 2004.

Japan: basic series							
	Output	Hours worked		Capital services			
				Exogenous RoR		Endogenous RoR	
	Volume index	Cost share	Volume index	Cost share	Volume index	Revenue share	Volume index
1985	100.0	73.61%	100.0	26.39%	100.0	29.35%	100.0
1986	103.0	75.51%	100.7	24.49%	104.4	29.98%	104.4
1987	106.9	77.02%	101.1	22.98%	109.3	30.28%	109.2
1988	114.1	77.74%	102.0	22.26%	115.2	31.72%	114.8
1989	120.1	77.24%	102.4	22.76%	122.2	32.42%	121.3
1990	126.4	73.80%	102.2	26.20%	129.1	32.60%	128.0
1991	130.6	73.72%	102.6	26.28%	136.4	32.19%	135.1
1992	131.9	72.24%	102.0	27.76%	143.2	32.16%	141.8
1993	132.2	72.20%	99.3	27.80%	149.2	31.81%	147.7
1994	133.7	72.84%	99.0	27.16%	154.4	31.49%	152.9
1995	136.2	72.40%	98.4	27.60%	160.2	31.23%	158.6
1996	140.9	71.80%	99.3	28.20%	167.2	32.28%	165.2
1997	143.5	70.92%	98.8	29.08%	174.4	32.22%	172.0
1998	141.9	69.20%	97.0	30.80%	181.0	31.91%	178.6
1999	142.0	71.00%	94.5	29.00%	187.3	32.11%	184.5
2000	146.0	71.88%	95.0	28.12%	192.9	32.60%	189.5
2001	146.7	69.41%	93.8	30.59%	197.2	32.61%	194.0
2002	146.1	71.57%	92.0	28.43%	198.9	32.80%	195.5
85-90	4.68%	75.82%	0.43%	24.18%	5.11%	31.06%	4.94%
90-95	1.50%	72.87%	-0.75%	27.13%	4.32%	31.91%	4.29%
95-00	1.38%	71.20%	-0.71%	28.80%	3.71%	32.06%	3.56%
95-02	1.00%	71.02%	-0.96%	28.98%	3.08%	32.22%	2.99%

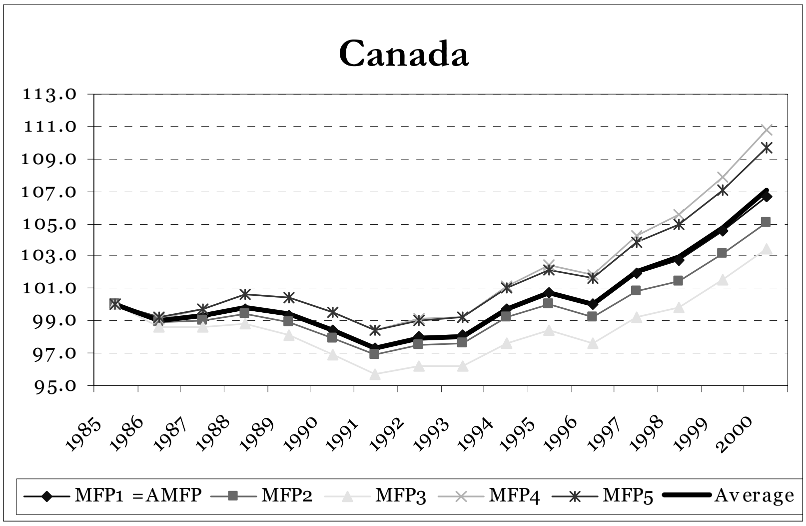
Japan: productivity measures						
	MFP1 =AMFP	MFP2	MFP3	MFP4	MFP5	Average
1985	100.0	100.0	100.0	100.0	100.0	100.0
1986	101.3	101.1	101.2	101.5	101.4	101.3
1987	103.7	103.3	103.4	104.3	103.9	103.7
1988	108.6	107.8	108.2	110.1	109.1	108.7
1989	112.5	111.1	111.8	114.8	113.2	112.7
1990	117.0	115.0	115.7	120.1	117.9	117.1
1991	118.9	116.4	117.2	122.4	119.9	118.9
1992	118.9	116.1	116.7	122.6	120.1	118.9
1993	120.2	117.1	117.4	123.9	121.3	119.9
1994	120.6	117.3	117.3	124.4	121.7	120.2
1995	122.3	118.7	118.8	126.2	123.4	121.8
1996	124.2	120.4	120.8	128.5	125.5	123.8
1997	125.3	121.3	121.7	129.8	126.7	124.9
1998	124.2	120.0	120.3	128.5	125.6	123.7
1999	125.2	120.9	121.1	129.6	126.6	124.6
2000	127.3	122.7	122.8	132.0	128.7	126.7
2001	128.1	123.4	123.3	132.9	129.6	127.4
2002	129.1	124.3	123.5	133.8	130.5	128.2
85-90	3.14%	2.79%	2.91%	3.67%	3.29%	3.16%
90-95	0.88%	0.63%	0.54%	0.99%	0.92%	0.79%
95-00	0.80%	0.68%	0.66%	0.89%	0.84%	0.77%
95-02	0.78%	0.66%	0.56%	0.84%	0.79%	0.72%

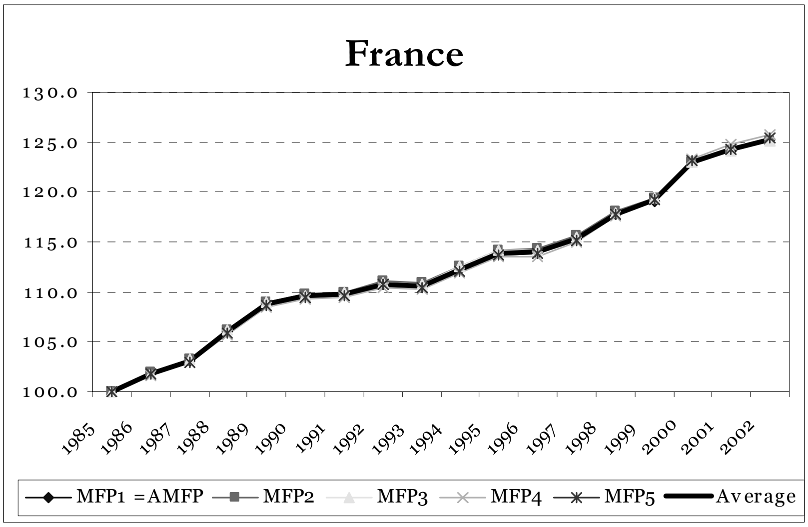
Source : OECD Productivity Database May 2004.

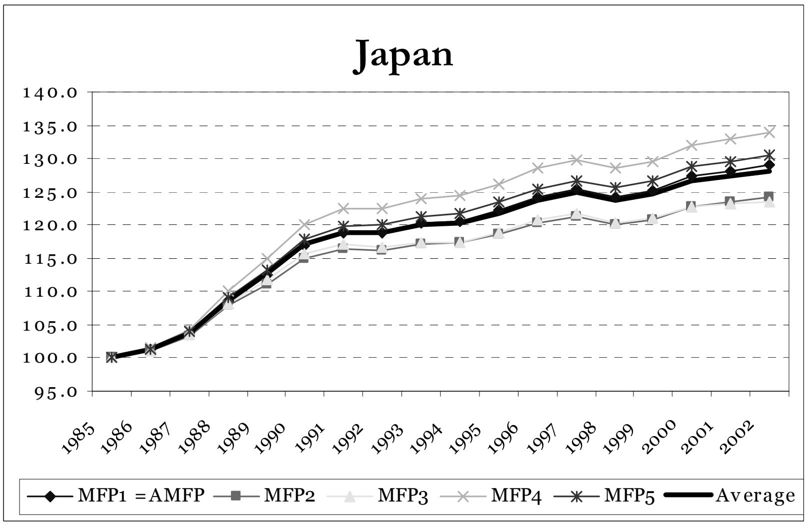
USA: basic series							
	Output	Hours worked		Capital services			
				Exogenous RoR		Endogenous RoR	
	Volume index	Cost share	Volume index	Cost share	Volume index	Revenue share	Volume index
1985	100.0	73.26%	100.0	26.74%	100.0	27.77%	100.0
1986	103.4	73.67%	101.1	26.33%	103.7	27.54%	103.7
1987	106.8	74.09%	104.0	25.91%	107.3	27.42%	107.1
1988	111.3	74.58%	107.1	25.42%	110.7	27.06%	110.5
1989	115.2	73.18%	110.0	26.82%	114.5	28.07%	114.2
1990	117.2	72.79%	110.7	27.21%	118.0	27.51%	117.7
1991	116.6	72.21%	108.9	27.79%	121.0	27.12%	120.7
1992	120.2	73.41%	109.2	26.59%	124.2	27.33%	123.9
1993	123.4	75.19%	112.0	24.81%	127.6	27.55%	127.1
1994	128.4	75.47%	114.8	24.53%	131.4	28.19%	130.7
1995	131.9	76.20%	117.4	23.80%	136.3	28.50%	135.2
1996	136.7	76.16%	119.0	23.84%	142.4	29.37%	140.6
1997	142.8	76.71%	122.3	23.29%	150.2	29.91%	147.3
1998	148.9	76.08%	125.3	23.92%	159.4	28.95%	155.1
1999	155.1	76.76%	127.7	23.24%	169.6	28.59%	163.8
2000	161.0	76.51%	129.4	23.49%	179.5	27.52%	172.4
2001	161.4	74.35%	128.4	25.65%	186.9	27.56%	179.1
2002	165.3	73.32%	127.8	26.68%	192.6	..	..
85-90	3.17%	73.60%	2.03%	26.40%	3.32%	27.56%	3.25%
90-95	2.36%	74.21%	1.18%	25.79%	2.88%	27.70%	2.78%
95-00	3.98%	76.40%	1.95%	23.60%	5.51%	28.81%	4.86%
95-02	3.22%	75.76%	1.21%	24.24%	4.94%	28.63%	..

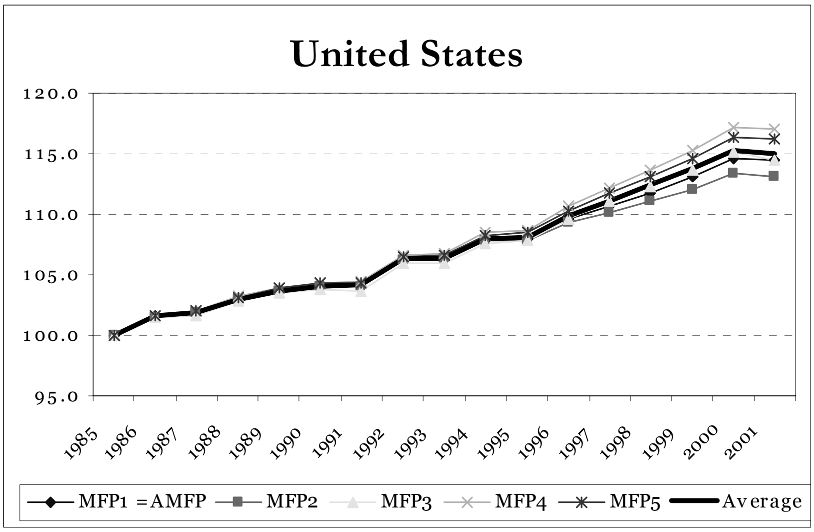
USA: productivity measures						
	MFP1 =AMFP	MFP2	MFP3	MFP4	MFP5	Average
1985	100.0	100.0	100.0	100.0	100.0	100.0
1986	101.6	101.6	101.5	101.6	101.6	101.6
1987	101.9	101.8	101.7	102.0	102.0	101.9
1988	103.0	103.0	102.8	103.2	103.2	103.0
1989	103.6	103.6	103.5	103.9	103.9	103.7
1990	104.1	104.0	103.8	104.4	104.3	104.1
1991	104.1	104.1	103.7	104.4	104.3	104.1
1992	106.3	106.2	105.9	106.6	106.5	106.3
1993	106.4	106.3	106.0	106.7	106.7	106.4
1994	107.9	107.8	107.6	108.4	108.3	108.0
1995	108.0	107.8	107.9	108.7	108.5	108.2
1996	109.6	109.3	109.6	110.6	110.3	109.9
1997	110.7	110.2	111.0	112.1	111.7	111.1
1998	111.8	111.1	112.4	113.7	113.1	112.4
1999	113.1	112.1	113.7	115.3	114.6	113.7
2000	114.6	113.4	115.1	117.2	116.4	115.3
2001	114.4	113.0	114.4	117.0	116.2	115.0
2002	116.7	..	..	..	..	116.7
85-90	0.80%	0.79%	0.75%	0.86%	0.84%	0.81%
90-95	0.73%	0.72%	0.76%	0.81%	0.79%	0.76%
95-00	1.19%	1.00%	1.30%	1.50%	1.39%	1.28%
95-02	1.11%	..	..	..	..	1.08%

Source : OECD Productivity Database May 2004.


Canada
113.0
111.0
109.0
107.0
105.0
103.0
101.0
99.0
97.0
95.0
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
MFP1 =AMFP
MFP2
MFP3
MFP4
MFP5
Average


France
130.0
125.0
120.0
115.0
110.0
105.0
100.0
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
MFP1 =AMFP
MFP2
MFP3
MFP4
MFP5
Average


Japan
140.0
135.0
130.0
125.0
120.0
115.0
110.0
105.0
100.0
95.0
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
MFP1 =AMFP
MFP2
MFP3
MFP4
MFP5
Average



7. Conclusions

This paper examines productivity and growth accounting measures when rates of return to capital inputs are exogenously determined. Several hypotheses about competition on output markets and about technology are invoked, each of which is compatible with exogenous rates of return. The following conclusions emerge:

The endogenous case – widely used in empirical research – imposes quite stringent assumptions: constant returns to scale and fully competitive output markets, all capital inputs are fully observed and there is perfect foresight by producers concerning expected changes in prices, and concerning rates of return and depreciation.

Different hypotheses entail different MFP measures. In the absence of further a-priori information or recourse to parametric techniques, there is no obvious way of discriminating between different hypotheses and to make an informed choice between productivity and growth accounting measures.

Empirically, the differences matter. This is evidenced for the four countries examined. In total, five different productivity measures were computed, each consistent with a particular set of assumptions.

The paper then goes on and suggests a pragmatic way forward – that of using an 'Apparent' MFP measure that is simply a ratio between a volume index of output and a volume index of observed aggregate inputs. It makes no claim to be a pure measure of technical change in the sense that it would represent a path-independent shift in the production or cost function. However, it is shown that AMFP

- is a measure that is close to the average of other measures;
- can easily be communicated due to its simplicity;

- can also be applied when assumptions about the nature of 'pure' technical change are relaxed, allowing, for example, for a formulation that encompasses neutral and biased technical change;

- relies on input measures that are independent from output measures and whose quality therefore does not vary with the quality and available detail of production or value-added data.

Clearly, the interpretation of AMFP has to be kept in mind: it reflects the combined effects of technical change, of non-observed inputs, of non-constant returns to scale and, indirectly, of deviations from perfect competition in product markets. In other words, AMFP is a true 'residual'. But for many practical purposes, it will fulfil its role as a multi-faceted measure of productivity growth.

References

Balk, Bert (1998) ; *Industrial Price, Quantity, and Productivity Indices*; Kluwer Academic Publishers.

Balk, Bert (2000); "Divisia Price and Quantity Indices: 75 Years After"; *Department of Statistical Methods, Statistics Netherlands.*

Basu, Susanto, John G. Fernald, Nicholas Oulton and Sylaja Srinivasan (2003); "The Case of the Missing Productivity Growth: or, does Information Technology Explain why Productivity Accelerated in the United States but not in the United Kingdom?"; *NBER Working Paper Series*, Working Paper 10010.

Basu, Susanto and Miles S. Kimball (1997); "Cyclical Productivity with Unobserved Input Variation"; *NBER Working Paper Series,* Working Paper No 5915.

Berndt, Ernst R. (1990); Comments on Hulten (1990).

Berndt, Ernst R. and Melvyn A. Fuss (1986); "Productivity Measurement with Adjustments for Variations in Capacity Utilisation and Other Form of Temporary Equilibria"; Journal of Econometrics, Vol.33.

Bureau of Labor Statistics (2003); Multi-factor productivity measures, available from http://www.bls.gov/mfp/home.htm

Bureau of Labor Statistics (1983); Trends in Multifactor Productivity, 1948-81, Bulletin 2178.

Christensen, Laurits R. and Dale W. Jorgenson (1969); "The Measurement of U.S. Real Capital Input, 1919-67"; Review of Income and Wealth, Series 15, No 4, pp 293-320.

Denny, Michael, Melvyn Fuss and Leonard Waverman (1981); "The Measurement and Interpretation of Total Factor Productivity in Regulated Industries with an Application to Canadian Telecommunications"; in T. Cowing and Stevenson (ed.); *Productivity Measurement in Regulated Industries*, Academic Press New York 179-218.

Diewert, Erwin W. (1980); "Aggregation Problems in the Measurement of Capital"; in Dan Usher (ed.), *The Measurement of Capital*, University of Chicago Press.

Diewert, Erwin W. (1983); "The Theory of the Output Price Index and the Measurement of Real Output Change"; Diewert and Montmarquette (eds.); *Price Level Measurement: Proceedings from a conference sponsored by Statistics Canada*, Ottawa.

Diewert, Erwin W. (2001); "Measuring the Price and Quantity of Capital Services under Alternative Assumptions"; *Department of Economics Working Paper* No 01-24, University of British Columbia.

Diewert, Erwin D. and Alice Nakamura (2005); "The Measurement of Aggregate Total Factor Productivity Growth"; forthcoming in Heckman and Leamer (eds.) *Handbook of Econometrics*, Volume 6.

Frisch, Ragnar (1936); "Annual Survey of General Economic Theory: the Problem of Index Numbers"; *Econometrica* 4, pp.1-39.

Fuss, Melvin and Daniel McFadden (eds.) (1978); *Production Economics: A Dual Approach to Theory and Applications*, Amsterdam, North Holland Publications.

Harper, Michael J., Ernst R. Berndt and David O. Wood (1989); "Rates of Return and Capital Aggregation Using Alternative Rental Prices"; in Dale W. Jorgenson and Ralph Landau (eds.); *Technology and Capital Formation*; MIT Press.

Hill, Peter and Robert Hill (2003); "Expectations, Capital Gains and Income"; *Economic Inquiry* 41, 607-619.

Hsieh, Chang-Tai (2002); "What Explains the Industrial Revolution in East Asia? Evidence from the Factor Markets"; *American Economic Review*, Volume 92, Issue 3, June, Pages 502-526.

Hulten, Charles R. (1986); "Productivity Change, Capacity Utilisation, and the Sources of Efficiency Growth", *Journal of Econometrics 33*.

Hulten, Charles (1990); "The Measurement of Capital"; in Ernst R. Berndt and Jack Triplett (eds.), *Fifty Years of Economic Measurement*, NBER.

Jorgenson, Dale W. and Kun-Young Yun (2001) ; *Investment Volume 3 Lifting the Burden: Tax Reform, the Cost of Capital and U.S. Economic Growth*, MIT Press.

Jorgenson, Dale W. and Zvi Griliches (1967) "The Explanation of Productivity Change"; *Review of Economic Studies* 34.

Oliveira Martins, Joaquim, Stefano Scarpetta and Dirk Pilat (1996); "Mark-up ratios in manufacturing industries - Estimates for 14 OECD countries"; *OECD Economics Department Working Papers* Nr 162.

Panzar, John C. (1989); "Technological Determinants of Firm and Industry Structure" in: Schmalensee, Richard and Robert D. Willig (eds.) *Handbook of Industrial Organization*; North Holland.

Paquet, Alain and Benoît Robidoux (2001); "Issues on the measurement of the Solow residual and the testing of its exogeneity: Evidence from Canada"; *Journal of Monetary Economics* 47; pp. 595-612.

Romer, Paul (1990); "Endogenous Technical Change"; *Journal of Politica*l Economy 98, pp. 71-102.

Samuelson, Paul A. (1953-54); "Prices of Factors and Goods in General Equilibrium"; *Review of Economic Studies* 21, 1-20.

Schreyer, Paul (2002); "Computer Price Indices and International Growth and Productivity Comparisons"; *Review of Income and Wealth*, series 48 Vol1, 15-33.

Schreyer, Paul (2001), "Information and Communication Technology and the Measurement of Volume Output and Final Demand – A Five Country Study", *Economics of Innovation and New Technology*, Vol 10.

Schreyer, Paul (2000); "The Contribution of Information and Communication Technology to Output Growth: A Study of the G7 Countries"; *OECD STI Working Paper*.

Schreyer, Paul, Emmanuel Bignon et Julien Dupont (2003); "OECD Capital Services Measures: Methodology and a First Set of Results"; *OECD Statistics Working Papers*.

Shephard, Ronald W. (1953); *Theory of Cost and Production Functions*. 1970 edition, Princeton, NJ: Princeton University Press.

Wyckoff, Andrew (1995); "The Impact of Computer Prices on International Comparisons of Productivity"; *Economics of Innovation and New Technology*, pp. 277-93.