0.1 A Note on the derivation of the model of "Advertising, Intangible Investment, and Unpriced Entertainment," Leonard I. Nakamura, June 1, 2004

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free entertainment goods with advertising

M is entertainment good

max

$U(z,M) = z + (b - a_u) M - \frac{1}{2}M^2$

$b, a_u > 0$, z is numeraire good,

s.t. $z + pM = y = 1 - M(c_M + \alpha') + pM \rightarrow z = 1 - M(c_M + \alpha')$

p is price of entertainment good

$\Lambda = z + (b - a_u) M - \frac{1}{2}M^2 - \lambda (z + pM - y)$

$\frac{d\Lambda}{dz} = 1 - \lambda = 0 \rightarrow \lambda = 1$

$\frac{d\Lambda}{dM} = b - a_u - M - \lambda p = 0 \rightarrow M = b - a_u - p$

Firm max:

firm set price p_M for good and gets α per unit sold from advertiser;

distribution cost per unit is $\alpha' + c_M$

Max $\Pi = (p + \alpha - \alpha' - c_M) M$

s.t. $M = b - a_u - p$

$\Pi = (p + \alpha - \alpha' - c_M)(b - a_u - p)$

$\frac{d\Pi}{dp} = b - a_u - p - (p + \alpha - \alpha' - c_M) = b - a_u + c_M - (\alpha - \alpha') - 2p = 0 \rightarrow$

$p = \frac{1}{2}(b - a_u + c_M - (\alpha - \alpha'))$

$\frac{d^2\Pi}{dp^2} = -2$

Thus if $\alpha - \alpha' > b - a_u + c_M$ then $p \leq 0$.

p=0$\rightarrow M = b - a_u$

$\Pi = (\alpha - \alpha' - c_M)(b - a_u) > (b - a_u)^2$

$U = z + (b - a_u) M - \frac{1}{2}M^2 = 1 - (b - a_u)(c_M + \alpha') + \frac{1}{2}(b - a_u)^2 = 1 + \frac{1}{2}(b - a_u)(b - a_u - c_M - \alpha')$

note: $(c_M + \alpha') < \alpha - (b - a_u) \rightarrow$

$U = 1 - (b - a_u)(c_M + \alpha') + \frac{1}{2}(b - a_u)^2 > 1 - (b - a_u)[\alpha - (b - a_u)] + \frac{1}{2}(b - a_u)^2 = 1 - \alpha (b - a_u) + \frac{3}{2}(b - a_u)^2$

Cost to advertisers: $\alpha M = \alpha (b - a_u)$

transmission cost: $c_M M = c_M (b - a_u)$

direct advertising cost $\alpha' M = \alpha' (b - a_u)$

value of entertainment $(\alpha - \alpha' - c_m)(b - a_u)$

entertainment subsidy $(\alpha - \alpha' - c_m)(b - a_u)\left(\frac{\alpha}{\alpha - c_M}\right)$

total advt cost $\alpha' (b - a_u)\left(\frac{\alpha}{\alpha - c_M}\right)$

direct utility gain: $(b - a_u) M - \frac{1}{2}M^2 = \frac{1}{2}(b - a_u)^2$

If advertising is absent, then:

$p = \frac{1}{2}\left(b + c_M\right)$
$M = \frac{b-c_M}{2}$
$\Pi = \left(p - c_M\right)\left(M\right) = \frac{1}{4}\left(b - c_M\right)^2$
$U(z, M) = z + bM - \frac{1}{2}M^2$
$z = 1 - Mc_M$

Direct utility gain: $bM - \frac{1}{2}M^2 = b\left(\frac{b-c_M}{2}\right) - \frac{1}{2}\left(\frac{b-c_M}{2}\right)^2 = \frac{(b-c_M)(4b-(b-c_M))}{8} = \frac{(b-c_M)(3b+c_M)}{8}$

Total utility gain: $z + bM - \frac{1}{2}M^2 = 1 - Mc_M + bM - \frac{1}{2}M^2 = 1 + \frac{1}{2}\left(b - c_M\right)^2 - \frac{1}{2}\left(\frac{b-c_M}{2}\right)^2$

$U = 1 + \left(b - c_M\right)\left(\frac{b-c_M}{2}\right) - \frac{1}{2}\left(\frac{b-c_M}{2}\right)^2 = 1 + \frac{3}{8}\left(b - c_M\right)^2$
cost to consumers: $pM = \frac{1}{4}\left(b^2 - c_M^2\right)$
transmission cost: $c_M M = c_M\left(b - c_M\right)$
payment to entertainer: $\frac{1}{4}\left(b - c_M\right)^2$