The Hedonic Regression Time-Dummy Method and the Monotonicity Axioms

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Abstract: This paper makes the point that the well-known and much applied hedonic regression time-dummy method, used to construct quality-adjusted price indexes, fails the monotonicity axioms from index theory. The paper discusses the hedonic time-dummy method and defines the monotonicity axioms in this context. A simple numerical example, using artificial data, is used to illustrate the failure of monotonicity. The reasons for this failure are identified and discussed. The frequency of the violation of monotonicity is considered for some particular cases and investigated for an actual data set. The paper concludes by considering the seriousness of the failure of monotonicity in this context and briefly looks at an alternative hedonic method that does satisfy monotonicity.

Key Words: Price Index, Quality Change, Hedonic Regression, Monotonicity Axioms.

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1. Introduction:

One common approach to constructing a quality-controlled price index is the hedonic regression *time-dummy* method, sometimes called the *direct* hedonic approach. Here a pooled regression is estimated with the logarithm of price explained by a set of quality characteristic and time-dummy variables. The price index is calculated directly from the time-dummy coefficients in the regression. As we will see this method has been frequently used by practitioners in the economic measurement literature to calculate price indexes for a diverse range of goods.

In this paper the validity of this approach is questioned in the context of its failure to satisfy the monotonicity axioms – an important set of axioms in the price index literature. These axioms are derived from the axiomatic or test approach to index numbers which treats an index formula as a function of independent variables and then specifies 'reasonable' properties that this function should possess. One of these reasonable properties is *monotonicity* of which there are two versions. The first states that the price index, which compares two periods, must increase if the later period's prices rise, holding other factors fixed. While the second states that the price index should decline if base prices are increased, holding other factors fixed. The satisfaction of the monotonicity axioms is regarded as a fundamental requirement for a price index to be a credible measure of price change.

In this paper it is shown that the hedonic regression time-dummy method of price index construction in fact does not satisfy these monotonicity requirements. This brings into question the use of this method to construct quality-adjusted price indexes.

In the next section the hedonic regression time-dummy method of index construction is discussed. Sections 3 and 4 outline the axiomatic approach to index numbers and define the monotonicity properties in the somewhat unusual context of the direct hedonic approach. Sections 5 gives a simple numerical example of the failure of monotonicity while Section 6 discusses the reasons behind this failure. Section 7 gives an empirical example, from actual data, of the failure of monotonicity. Sections 8 and 9 discuss the economics of the failure of monotonicity and suggest alternatives while Section 10 concludes.

2. The Hedonic Regression Time-Dummy Index Calculation Method:

To use the hedonic regression time-dummy approach for the construction of price indexes you must specify and estimate a hedonic regression. This hedonic regression pools cross-sectional data, on the prices and quality characteristics for a good, over time. For some good of interest we have the following hedonic regression model.

$$\ln(p_i^t) = \alpha + \sum_{k=1}^{K} \beta_k z_{i,k}^t + \sum_{\tau=2}^{T} \delta_\tau d_{i,\tau}^t + \varepsilon_i^t, \quad i = 1,...,N, \; t = 1,...,T \tag{1}$$

The set of observations on goods is indexed $i = 1,2,...,N$, and the time period by $t = 1,...,T$. However, perhaps not all goods appear in each time period. In the model the logarithm of price, $\ln(p_i^t)$, is explained by an intercept, a set of quality-characteristics, $z_{i,k}^t \;\; k = 1,2,...,K$, and time-dummy variables, $d_{i,\tau}^t \;\; \tau = 1,2,...,T$.

After estimating this regression the quality-adjusted price index can be calculated very simply and directly by taking the exponential of the time-dummy coefficients of interest. The justification for this is straightforward. If we compare the relative price of a good, between say period t and period s, for any given quality configuration, say represented by $\bar{z}$, then this ratio is equal to the relative exponential of the time dummy variables. This is shown in (2).[1]

$$P_{TD}^{t,s} = \frac{\hat{p}^t(\bar{z})}{\hat{p}^s(\bar{z})} = \frac{\exp\left(\hat{\alpha} + \sum_{k=1}^{K} \hat{\beta}_k \bar{z}_{i,k} + \hat{\delta}_t\right)}{\exp\left(\hat{\alpha} + \sum_{k=1}^{K} \hat{\beta}_k \bar{z}_{i,k} + \hat{\delta}_s\right)} = \frac{\exp(\hat{\delta}_t)}{\exp(\hat{\delta}_s)} = \exp(\hat{\delta}_t - \hat{\delta}_s) \qquad (2)$$

The simplest case is where s is the base period, period 1 in this case, which has been normalised to zero. Then the estimated price index for period t relative to the base period is simply the exponential of the time-dummy coefficient for period t. An important and desirable feature of this index method is that it does not depend upon the particular value of the quality-characteristics vector chosen ($\bar{z}$).[2]

This method for constructing quality-adjusted price indexes is widely accepted in the theoretical index literature. The first person to use this method was Court (1939) examining automobile prices however the method was rediscovered and popularized by Griliches (1961).[3] This method has a long pedigree and while other hedonic methods have since been developed the time-dummy method still remains important. For instance, in a recent paper Diewert (2001b) outlined a consumer theory hedonic model where the index could be determined using the time-dummy method.

The use of this particular hedonic technique for price index construction is widespread in the empirical literature, often amongst other hedonic techniques. A good discussion and some examples can be seen in Gordon (1990) who applies the method to various durable goods. Berndt, Griliches and Rappaport (1995) used the method for looking at personal computer prices in the US in the 1990s and found very large price

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[1] It is known in the econometrics literature that exponential of the time dummy variable in (2) is a biased estimate of the equivalent population parameter as the estimated coefficient is random while the transformation is nonlinear. This has to some extent been neglected in much of the empirical applications of the hedonic regression time-dummy approach. However, in this paper we will not dwell on this problem as it does not influence any of our main points. The literature and the solution to the problem are most recently discussed in van Garderen and Shah (2002).

[2] Note also that we would get the same results if the intercept was excluded from the model (1) and a time-dummy for every time period was included. In the regression *without an intercept* the estimated time-dummy parameter for period t would be equal to $\hat{\delta}^t + \hat{\alpha}$, where $\hat{\delta}^t$ is the time-dummy coefficient *with an intercept* included. As we are just adding a constant to each of the estimated time-dummy parameters in the case without an intercept these cancel when looking at relative exponents. Hence the index is unchanged.

[3] The history of hedonic regression is discussed in greater detail in Berndt (1991).

declines. There are numerous other applications of the direct hedonic method to computers, these include; ABS (2001), Nelson, Tanguay and Patterson (1994), Shiratsuka (1995) and Okamoto and Sato (2001) who look at PCs as well as TVs and Digital Cameras. There are also applications of the time-dummy approach in other areas including; Silver (1999) and Silver and Heravi (2001) who look at television prices in the UK and Kokoski, Waehrer and Rozaklis (2001) who look at consumer audio products. No doubt there are many other such applications which have escaped the author's attention.

However, some theoreticians have objected to the time-dummy technique and have focused on alternative hedonic approaches (Silver, Ioannidis and Webb, 2001). One popular alternative is the *imputation method*. Generally speaking, this involves estimating a hedonic price function each period and using this function to impute estimated prices for the non-matched models. These prices are then used in an index formula in a conventional manner. This procedure is firmly nested within conventional index methods so does not represent an alternative index calculation method, as does the time-dummy method outlined above.

The adoption of hedonic methods by statistical agencies has been particularly rapid in the past decade. Moulton (2001, p.1) noted that,

> …currently [in the US] 18 percent of the final expenditures in gross domestic product is deflated using price indexes that use hedonic methods.

The most common implementation of hedonic methods involves the use of the imputation method. However, in the United States, some official indexes are based upon the time-dummy hedonic method. Moulton (2001, p.2) writes,

> …the Federal Reserve's indexes for LAN routers and switches and the hedonic portion of the BEA [Bureau of Economic Analysis] software index… are calculated from the regression coefficients of indicator (or "dummy") variables for years…

From my review of websites and official papers it is unclear to what extent other countries statistical agencies have adopted the time-dummy hedonic method to produce official statistics. However, a number of countries have used the time-dummy method to investigate the effect of adopting hedonic methods (ABS, 2001; Kokoski, Waehrer and Rozaklis, 2001).

3. The Axiomatic Approach to Index Numbers and the Time-Dummy Method:

Given the time-dummy method for constructing hedonic-type price indexes it is interesting to know how this method of index construction performs with regard to the *test* or *axiomatic* approach to index numbers. This approach to the construction of index numbers has a long history and dates back at least until the start of the twentieth century. It is best illustrated by Irving Fisher (1922) in his classic book, *The Making of Index Numbers*.[4]

[4] For a useful set of references and discussion outlining the axiomatic approach see Diewert (2001a).

The axiomatic approach treats an index formula as a function of independent variables and specifies 'reasonable' properties or tests that this function *should* have with regard to these variables. One advantage of this approach is that it does not depend upon the assumption of optimizing agents which is fundamental to the economic approach to index numbers (Diewert, 1976). There is a long list of axioms which are often used to test index formulae (Diewert, 1992). In this paper the focus is on a single set of axioms, the *monotonicity properties*. These properties are rigorously defined in a moment. First however, it is useful to compare at a general level the hedonic regression time-dummy method for constructing a price index with more conventional methods.

Most price index formulae are a function of the vectors of base and current period prices and quantities. Thus a conventional price index can be written as the following general function where p^r and q^r are price and quantity vectors for periods $r = t, t-1$.

$$P = I(p^t, p^{t-1}, q^t, q^{t-1}) \quad (3)$$

If there are N goods in each period then a conventional index takes $4N$ positive real numbers and transforms them into a single positive real number – the index value.[5]

Unlike conventional price indexes the hedonic regression time-dummy method is a function of the prices of at least two periods as well as the *quality-characteristics* of these goods in each period. Here let us consider the case where only two periods are compared (in regression (1) above $T = 2$). Let us also fix the number of goods for which we have prices each period at N.[6] Both these simplifications are in order to ease the exposition and they do not diminish any of the points made. Given this, we can write the general form of the hedonic regression time-dummy index (P_{TD}) in the following way.

$$P_{TD} = I(p^t, p^{t-1}, Z^t, Z^{t-1}) \quad (4)$$

Here we pool data over just two time periods t and $t-1$ and use Z^r to denote the matrix, of dimension $N \times K$, of quality characteristics for the goods in period $r = t, t-1$. It can be seen that the hedonic regression time-dummy method, in the case of two time periods, transforms a set of $2 \times N + 2 \times (N \times K)$ variables into a single positive real number. With this definition for the hedonic regression time-dummy method we can proceed to defining the monotonicity axioms.

[5] A price index may be a function of more than 4N variables if, for example, expenditure weights are derived from many periods. Also a price index can be a function of less than 4N variables, for example, the Laspeyres price index uses base quantities but not current quantities so is a function of just 3N variables.

[6] In general the number of goods used to construct the hedonic regression price index can change over time without causing problems. This is one of the benefits of the hedonic approach. Changing sets of goods poses greater problems for conventional price index methods.

4. The Monotonicity Axioms for the Hedonic Regression Time-Dummy Method:

There are two monotonicity axioms for price indexes. Diewert (2001a) identifies Eichhorn and Voeller (1976) as the first to suggest these axioms and they are regarded as fundamental in the price index literature. The first is called the *Monotonicity in Current Prices Axiom* and is shown in (5).

$$I(p_X^t, p^{t-1}, Z^t, Z^{t-1}) > I(p^t, p^{t-1}, Z^t, Z^{t-1}), \quad \text{if } p_X^t > p^t \tag{5}$$

Here $p_X^t > p^t$ means that each element of the vector p_X^t is no smaller than the corresponding element of p^t and there is at least one element in p_X^t which is strictly larger than the corresponding element in p^t. In words (5) states that if the current price vector increases, holding other variables constant, then the index must also increase.

The base period twin to (5) is the other monotonicity axiom, (6) below, and is called the *Monotonicity in Base Prices Axiom*.

$$I(p^t, p_X^{t-1}, Z^t, Z^{t-1}) < I(p^t, p^{t-1}, Z^t, Z^{t-1}), \quad \text{if } p_X^{t-1} > p^{t-1} \tag{6}$$

While these are the two versions of the monotonicity axioms that have been most widely used we could perhaps weaken these axioms somewhat. The reason being that we may not expect strict equality, as in (5) and (6), if for example the good for which the price has increased did not sell on the market. This leads us to consider the case where the index value may not change when prices increase. Then (5) becomes the *Weak Monotonicity in Current Prices Axiom* below.

$$I(p_X^t, p^{t-1}, Z^t, Z^{t-1}) \geq I(p^t, p^{t-1}, Z^t, Z^{t-1}), \quad \text{if } p_X^t > p^t \tag{7}$$

Similarly for base prices (6) becomes the *Weak Monotonicity in Base Prices Axiom* below.

$$I(p^t, p_X^{t-1}, Z^t, Z^{t-1}) \leq I(p^t, p^{t-1}, Z^t, Z^{t-1}), \quad \text{if } p_X^{t-1} > p^{t-1} \tag{8}$$

These four monotonicity axioms are intuitively appealing properties for price indexes to possess. Many regard the monotonicity axioms as a fundamental requirement for a price index to be credible and the failure of an index to satisfy these axioms as a serious shortcoming of the method. In Balk's (1995, p.70) survey of the axiomatic approach to index numbers he distinguishes between "…axioms – which are more or less self-evident – and tests – about which more debate is possible". The monotonicity property is the *first axiom* that Balk reports. Indeed, as Reinsdorf and Dorfman (1999) have noted, it would be a daunting task to have to explain to an index user why a price index should be considered for use that does not satisfy the monotonicity axioms.

However, in the next section, it is shown that the hedonic regression time-dummy method of index construction does not generally satisfy these axioms.

5. A Simple Numerical Example:

Let us consider a simple numerical example of the failure of the *Monotonicity in Current Prices Axiom* using artificial data. Suppose that we have five observations over two time periods, t and $t-1$, with two price series; prices *A*; p_A^t and p_A^{t-1}, and prices *B*; p_B^t and p_B^{t-1}. In order to test monotonicity in current prices we have set $p_B^t > p_A^t$ with the only difference between them being the final observation, which is strictly larger for p_B^t than for p_A^t. The base period prices for series *A* and *B* have been kept fixed, $p_B^{t-1} = p_A^{t-1}$. Suppose we also have information on the value of a quality variable. This data along with the time-dummy variable is shown in Table 1 below.

Table 1: Data.

Observation Number	Time Period	Prices A (p_A)	Prices B (p_B)	Quality Variable	Time-Dummy Variable
1	$t-1$	2	2	2	0
2	$t-1$	5	5	4	0
3	t	6	6	4	1
4	t	8	8	5	1
5	t	9	*10*	6	1

To test *Monotonicity in Current Prices* we adopt the regression procedure, outlined in Section 2 above, first using the *A* price series then using the *B* price series, and compare the resulting price indexes. We estimate a regression of the logarithm of price upon the quality parameter, a time-dummy and an intercept with the results being shown in Table 2 below.

Table 2: Results.

Coefficients	Estimates A	Estimates B
Intercept	0.1600	0.0810
Quality	0.3304	0.3568
Time-Dummy	0.2106	0.1931

Using this information we can straightforwardly estimate the price index as the exponential of the time-dummy parameter estimate. The indexes are shown in Table 3.

Table 3: Price Indexes.

Index	Value
Index A	1.2344
Index B	1.2130

As can be seen in Table 3, we have the somewhat perplexing result that the price index for period t vs. $t-1$ derived from prices B is actually *lower* than that derived from prices A. This simple example shows that in general price indexes constructed using the hedonic regression time-dummy method are non-monotonic (or even weakly monotonic) in current prices. The axioms (5) and (7) above are violated. We could also construct an analogous example to illustrate that the index method is also not monotonically decreasing in base prices.

In the simple example above the price index decreased even though the later period price vector increased. In theory it is possible that the regression coefficient on the time-dummy variable can increase, decrease or remain constant, as current prices increase. In the next section we take a more detailed look at how the coefficient is estimated and why monotonicity fails.

6. Monotonicity and Regression Coefficients:

The results above indicate that the monotonicity axiom is violated by the hedonic regression time-dummy method. Considering the case more generally we can illustrate how this problem arises.

Let us rewrite the hedonic regression in equation (1) above in matrix notation. Here let y be the $(T \times N) \times 1$ vector of the dependent variable, the logarithm of the observations on price over at minimum two periods, $T \geq 2$. The full matrix of quality characteristics is represented by Z and has dimensions $(T \times N) \times K$. We make the intercept in the model explicit with e being a vector of ones of length $T \times N$. D is the time-dummy vector (or matrix in the case where we have more than two time periods) and has dimensions $(T \times N) \times (T-1)$. The estimators are represented by the vector α for the intercept, β for the quality variables, and δ for the time-dummy variables.

$$y = e\alpha + Z\beta + D\delta + \varepsilon \qquad (9)$$

Let us simplify the discussion a little by again assuming just two periods are compared in regression (9) (i.e. $T = 2$ and D is a vector of length $2N$). We can use a fundamental result in econometrics, the Frisch-Waugh Theorem, to write the coefficient estimate of the time dummy in a very simple way.

The Frisch-Waugh Theorem tells us that the estimated time-dummy coefficient, $\hat{\delta}$ in (9), can be obtained by first estimating a regression of the quality-characteristics and intercept on the time-dummy variable, such as in (10) below, and then taking the residuals from this regression, u, and regressing the dependent variable y on these residuals.

$$D = e\psi + Z\gamma + u \tag{10}$$

Then using the Frisch-Waugh Theorem we can write the estimated coefficient on the time-dummy variable in the following way.[7]

$$\hat{\delta} = (D^T MD)^{-1} D^T My \tag{11}$$

$$= ((MD)^T (MD))^{-1} (MD)^T y \tag{12}$$

$$= (u^T u)^{-1} u^T y \tag{13}$$

The usefulness of (13) comes from the fact that it is linear in the dependent variable. Let us differentiate (13) with respect to a particular observation $i = 1,...,N$ from period $r = t, t-1$. This gives the exact change in the coefficient for a change in the dependent variable due to the linearity of the expression.

$$\frac{d\hat{\delta}}{dy_i^r} = \frac{u_i^r}{u^T u} \tag{14}$$

From (14) it is clear that if u_i^r, the error for good i in period r of the regression (10), is negative then the time-dummy coefficient will be decreasing in terms of that particular observation of the dependent variable while if the error is positive it will be increasing in terms of that observation. This is because the denominator of (14) is always non-negative, $u^T u \geq 0$, and it will in fact be positive as long as the usual assumption, that there are no perfect linear relationships between the dependent variables, is satisfied in the original regression model (9).

Then when we have just two periods, the current period t and the base period $t-1$, we have the following condition for *Monotonicity in Current Prices* to fail.

$$u_i^t < 0 \quad \Rightarrow \quad \frac{d\hat{\delta}}{dy_i^t} < 0 \quad \Rightarrow \quad \textit{Monotonicity in Current Prices Fails.} \tag{15}$$

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[7] Here we define the matrix $X \equiv [e \quad Z]$ and $M = (I - X(X^T X)^{-1} X^T)$ is the 'annihilator' or 'residual maker' matrix which is both symmetric ($M = M^T$) and idempotent ($M^2 = M$), Greene (2000). Note that equation (13) follows from (12) as the matrix operation MD gives the residuals of the regression (10).

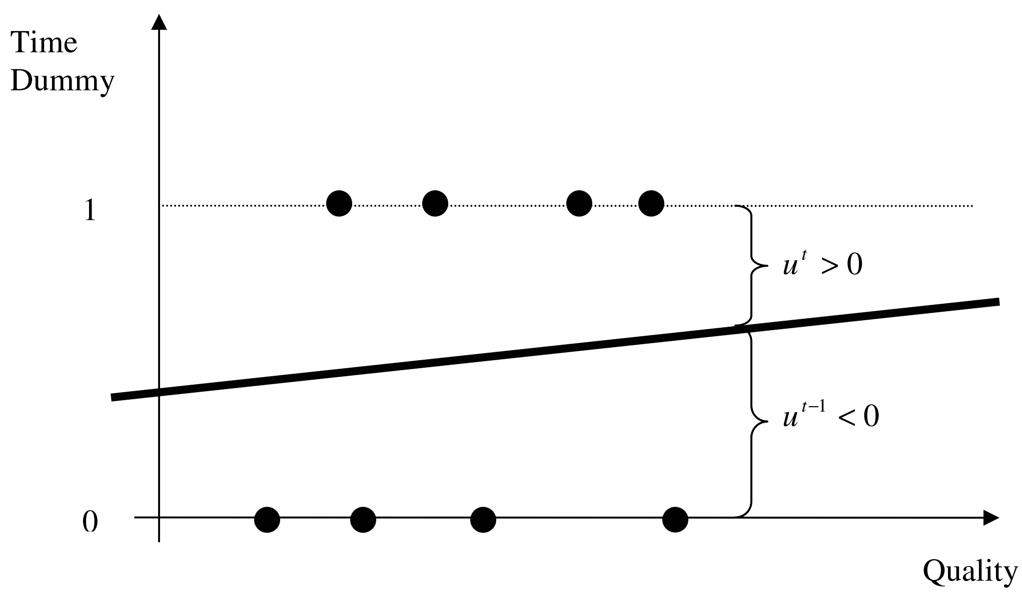
We can construct a similar condition for the failure of *Monotonicity in Base Prices*.

$$u_i^{t-1} > 0 \quad \Rightarrow \quad \frac{d\hat{\delta}}{dy_i^t} > 0 \quad \Rightarrow \quad \textit{Monotonicity in Base Prices Fails.} \tag{16}$$

That is, for monotonicity to *fail*, we require that some of the error terms in the regression (10) are negative for current-period observations and positive for base-period observations. Clearly, as the previous section showed, it is possible to construct examples where monotonicity fails but can more be said about the likelihood of this failure? We consider two cases below and start by considering a case where monotonicity does not fail.

(i). Suppose that there is no relationship of any significance between the quality variables and the time-dummy variable. Then the estimated coefficients in the auxiliary regression (10) above will be close to zero and the regression intercept will be close to the average of the time-dummy variables. Calculating the error terms from the regression we will find that the errors for all current observations are positive while those for the base period are negative as the average of the time-dummy is between zero and one. Here monotonicity will not be violated. This is depicted in Figure 1 below for the case where there is a single quality parameter.

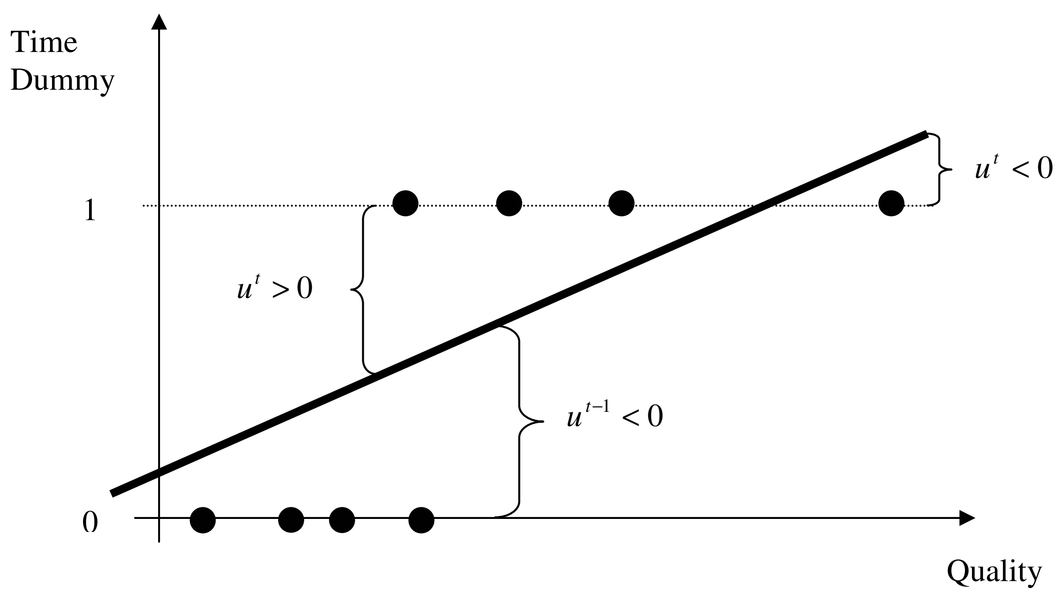
Figure 1: Quality Independent of Time.



Note that the case depicted in Figure 1 seems somewhat unlikely. Remember that the primary reason that we use the hedonic regression technique to estimate price indexes is because there are changes in the quality of goods over time. If the quality of goods were unchanging through time then it would be relatively easy to match prices for similar quality goods over time and we could apply traditional index methods. In this scenario a hedonic regression would seem unwarranted. Therefore it seems unlikely that in most of the cases where the hedonic regression time-dummy method is used that the quality characteristics and time would be unrelated. Let us now consider another case of interest where monotonicity fails.

(ii). Suppose that there is now some linear relationship between the time-dummy variable and the quality characteristics, which will be reflected in regression (10). In general, it is possible that any particular pattern of errors could result. In the regression (10) monotonicity will fail if a current period observation lies below the regression hyperplane or a base period observation lies above the hyperplane. In Figure 2 below we have depicted the case, for just one quality-characteristic, where the error for one observation from the regression (10) lies below the regression line for the later period t (i.e. $u^t < 0$). This implies that *Monotonicity in Current Prices* fails for this observation.

Figure 2: Quality Dependent on Time.



Note that our simple numerical example in Section 5 was of the type depicted in Figure 2. As can be seen from Figure 2 one of the preconditions for the failure of monotonicity is that a relationship exists between the time-dummy variables and the quality characteristics. In practice it is difficult to predict the extent of such a relation and hence the extent of non-monotonicity. To investigate this further we turn to an empirical investigation.

7. An Empirical Example:

In this empirical example we use data from the classic Cole *et al* (1986) study into computer prices. In particular we use the corrected data on *hard disk drives* available in the textbook by Ernst Berndt (1991), *The Practice of Econometrics: Classic and Contemporary*. This data is composed of 91 observations of list prices on 30 devices marketed by 10 vendors from 1972 to 1984 in the US (Cole *et al*, 1986, p.44). As well as price (p) there are a number of quality-characteristics, however, in constructing the hedonic regression we use the formulation of Cole *et al* (1986) who use two quality measures. Their quality-characteristics are, firstly, the *capacity* (c) of the hard drive in megabytes and secondly, a derived measure of the *speed* (s) of the hard drive in transferring one kilobyte of data. In the succeeding discussion we use this basic data to look at the frequency of the failure of monotonicity.

One of the primary models that Cole *et al* (1986) considered, model *I* in their paper, was a pooled regression over all time periods. The model is shown below where we have normalized δ^{1972} to zero. Here the time periods run from $t = 1972, 1973, ..., 1984$ but the number of goods each period varies. The model is shown in (17) while the results of this regression, for those interested, are reported in Table 4 of the Appendix. The resulting price indexes are shown in Table 6.

$$\ln(p_i^t) = \alpha + \beta_c \ln(c_i^t) + \beta_s \ln(s_i^t) + \sum_{t=1973}^{1984} \delta^t d_i^t \qquad (17)$$

The discussion thus far has focused on the two-period time-dummy regressions whereas here we have a multi-period regression. However, we can easily generalize the discussion, and our method for finding non-monotonicity, to the multi-period case. In equation (10) for the two-period case, in order to determine the 'weights' that the various observations of the dependent variable receive in estimating the time-dummy parameter, we ran a regression of the single time-dummy variable on the quality-characteristics. However, in the multi-period case we instead run a regression of the time-dummy variable of interest on the quality characteristics as well as the remaining time-dummy variables. The errors from this regression can be given the same interpretation as in (14) above and provide a ready mechanism for checking monotonicity.

Now turning back to the empirical discussion, as we have made 1972 our base period in (17) each of the time-dummy variables measures the *difference* in log-price level from 1972. This means that for *Monotonicity in Current Prices* to *fail*, for say $\hat{\delta}^{1980}$, we require that $\hat{\delta}^{1980}$ be a decreasing function of price observations from 1980 and for *Monotonicity in Base Prices* to *fail* we require that $\hat{\delta}^{1980}$ be an increasing function of price observations from 1972. Using this method and the model in (17) only a single violation of monotonicity was detected. This was for an observation in 1972 where an *increase* in the price for this observation would have *increased* the estimated time-dummy coefficient for 1984.

Let us make further use of the data and, rather than doing a pooled time-dummy regression, let us undertake a series of adjacent two-period regressions and check for monotonicity. In this case we use the following regression model over two adjacent years where we insert a time-dummy variable for the later period.

$$\ln(p_i^t) = \alpha + \beta_c \ln(c_i^t) + \beta_s \ln(s_i^t) + \delta^t d_i^t \qquad (18)$$

In total this involved twelve regressions. Again, for those interested the, results of this regression are shown in Table 5 of the Appendix with the price indexes shown in Table 6.[8] Using the method exactly as detailed in Section 6 above we were able to determine that *Monotonicity in Base Prices* failed in the 1977–78 comparison for 2 observations in 1977. Here the time-dummy variable for 1978 would have *increased* if the prices for these two observations in 1977 had in fact *increased*. *Monotonicity in Current Prices* failed in a single case for the 1981–80 comparison where the time-dummy coefficient for 1981 would have *decreased* if a price for 1981 had *increased*.

While the failure of the monotonicity axiom does not seem that prevalent, at least in this particular data set, any failure of monotonicity seems a serious shortcoming of the method. In the next section we consider how the hedonic regression time-dummy methods failure of monotonicity should be regarded.

8. Discussion of the Failure of Monotonicity:

The previous sections have outlined and illustrated the failure of the monotonicity axioms for the hedonic regression time-dummy method. An important question then is whether the failure of the monotonicity axiom is a serious shortcoming of this method.

There are a number of well-used indexes that also fail the monotonicity axioms. The most notable casualty is the Tornqvist Index. This and the failure of the Sato-Vartia index and the Geometric Mean Index are discussed in Reinsdorf and Dorfman (1999). An interesting point made by Reinsdorf and Dorfman (1999) is that the failure of these indexes to satisfy the monotonicity axioms may not be a fault of the indexes themselves but of the axiom being applied in an economic context. Let us consider their argument.

In addressing the question of the importance of the failure of monotonicity it is useful to discuss the economics of the monotonicity axioms. The axiomatic approach to index numbers stands outside the economic approach – nevertheless the monotonicity axioms can be given a strong economic interpretation.

Consider a cost function, defined below as the minimum cost of reaching a particular level of utility, $\overline{U}$, given the price vector, p .

$$C(p,\overline{U}) \equiv \min{}_x \{ p^T x : U(x) \geq \overline{U} \} \qquad (19)$$

[8] These adjacent period regressions clearly have some problems. In some cases the sign of the coefficients of speed and capacity are negative when we would have a strong a priori expectation that the coefficients should be positive. However, these regressions are used for illustrative purposes only.

Now if there are two price vectors, call them p_A and p_B, and $p_B > p_A$ in the sense defined earlier. Then the following inequality must hold.[9]

$$C(p_B,\overline{U}) \geq C(p_A,\overline{U}) \qquad (20)$$

The economic approach to index numbers defines a price index (i.e. a cost of living index) as the relative cost of reaching a given level of utility under two different price regimes (Diewert, 1976). Dividing (20) through by some common base period cost function we arrive at (21).

$$I(p_B, p_0,\overline{U}) \equiv \frac{C(p_B,\overline{U})}{C(p_0,\overline{U})} \geq \frac{C(p_A,\overline{U})}{C(p_0,\overline{U})} \equiv I(p_A, p_0,\overline{U}) \qquad (21)$$

It can be seen that an economic price index *must* satisfy the *economic version* of the *Weak Monotonicity in Current Prices Axiom*. With similar reasoning we could of course provide an analogous relationship for the *Weak Monotonicity in Base Prices Axiom*.

However, it is important to note that an economic index fixes preferences and the required utility level while the quantities of goods consumed are endogenous. But the conventional formulation of the monotonicity axioms takes quantities as one of the fixed variables (see (3) above). Generally, then a test of the monotonicity of an index formula will not be a test of the monotonicity property in the economic sense. This was the problem identified by Reinsdorf and Dorfman (1999) with regard to the Tornqvist, Sato-Vartia and Geometric Mean indexes. It was for this reason that Reinsdorf and Dorfman (1999) questioned the monotonicity axiom. In the case of the Sato-Vartia index Reinsdorf and Dorfman (1999, p.57) write,

> If the goal is to estimate a cost of living index, it is the monotonicity axiom rather than the Sato-Vartia index that is suspect. By letting quantities make the same change when prices change to p_1 as they do when prices change to p_1^*, the monotonicity axiom implicitly assumes that commodities are more substitutable in one case than in the other [i.e. preferences are different in the two comparisons].

The essence of their argument is that the economic application of monotonicity, which fixes preferences, is not the same as the way in which monotonicity is conventionally

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[9] To see that this inequality holds consider the following argument. Suppose that x_A is the bundle that satisfies the cost minimization problem (19) when prices are equal to p_A and similarly x_B is optimal given p_B, where $U(x_A) = U(x_B) = \overline{U}$. Then the following chain of inequalities must hold: $C(p_B,\overline{U}) = p_B^T x_B \geq p_A^T x_B \geq p_A^T x_A = C(p_A,\overline{U})$. The first inequality holds as $p_B > p_A$ and the second inequality holds as x_A is optimal given p_A.

specified in that the monotonicity axioms fix quantities. How does this point relate to our finding that the time-dummy hedonic method fails monotonicity?[10]

The economic theory of hedonic regressions is well developed though it is far more contentious than the economic theory of index numbers (Rosin, 1974; Feenstra, 1994; Diewert, 2001b; Pakes, 2002). However, much can be gleaned from this literature. In terms of the hedonic approach to index numbers the requirement that seems equivalent to *keeping preferences constant* is that *the value placed upon the quality-characteristics does not change*. In the economic models of hedonic regression the estimated coefficients represent the value that the market places upon the various characteristics in terms of both the cost that firms face in producing the characteristics and the value that buyers place upon the characteristics. When the value placed upon characteristics changes then implicitly the market conditions must also have changed. However, what we have done in testing monotonicity is to allow the value of characteristics to change when we changed a price. We have re-estimated the hedonic function with the new prices and this has naturally led to changes in the assessment of the value of characteristics as well as changes in the time-dummy coefficients. This is just what happened in our numerical example in Section 5. Here we raised a later-period price and this rise was ascribed to a change in the assessment of the value of the characteristics and the intercept rather than the time-dummy variable. However, suppose instead that we had imposed the requirement that the features of the market were unchanged which we have argued in the case of hedonic regression means fixing the value placed upon characteristics. By not allowing the assessment of the characteristics to change the only coefficient that can change is that for the time-dummy variable. In this case clearly the monotonicity axioms will be satisfied – the time dummy will move in the same direction as the changes in the dependent variable.

From an economic perspective then perhaps the failure of monotonicity axioms is not as serious as it first appears. It is interesting however that we must fall back upon an economic perspective to justify the time-dummy hedonic method. It also raises a pressing question about the time-dummy hedonic regression procedure. The problem of the failure of monotonicity arises because the time-dummy hedonic method endeavors to simultaneously estimate the market relation between prices and characteristics, reflecting consumer preferences and producer costs, and also to estimate the inflation factor, reflected in the time-dummy variable. Looking at the procedure this way it is not so surprising that we can fool the hedonic time-dummy method into thinking that the value of characteristics has changed when in fact it is the price level that has changed.

Despite the economic explanation for the failure of monotonicity the fact still remains that this property seems extremely important from an intuitive perspective. That is we can construct examples, as we have above, where the price index produced by the hedonic time-dummy method produces perverse, anti-intuitive and even absurd results. In this case it seems warranted to explore other hedonic methods where monotonicity will not fail. We briefly outline such a method in the next section.

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[10] The discussion in the following paragraphs of this Section was greatly improved by comments from a referee.

9. An Alternative Hedonic Regression Method:

There is one appealing alternative that deserves strong consideration because it satisfies monotonicity and that preserves the basic idea of calculating an index directly from estimated regression coefficients. This is called the *generalised dummy variable method* and is outlined in Diewert (2001b). Here the quality-characteristics space of the good is partitioned into categories and a dummy variable is used to represent each category in a regression equation.[11] In addition to a single category dummy for each price observation a time-dummy variable is included. It is straightforward to see that in this case, where we have a time-dummy variable and a *single* quality-dummy variable, the regression of the time-dummy variable on the quality-dummy variable with an intercept (i.e. equation (10) above) *must* produce non-negative errors for current period observations and non-positive errors for base period observations. The reason for this is that the regression of a zero-one time-dummy on a zero-one quality-dummy must pass below (above) all current (base) period observations. Hence monotonicity is satisfied for this method.

Interestingly the Country Product Dummy (CPD) method (Summers, 1973) is a special case of the generalized dummy variable method that uses a single quality dummy-variable approach in making spatial (as opposed to intertemporal) comparisons of prices. For this reason the CPD method does not fail the monotonicity axioms discussed above.

Diewert (2001b) discusses how to implement the generalized dummy variable method in more detail. To the best of my knowledge this approach has not been used in the hedonic literature on intertemporal price comparisons. This method has the advantage of satisfying the monotonicity property as well as great flexibility in the functional form of the regression. One major drawback is that it is rather demanding of the data. Firstly, there are likely to be a large number of characteristics-configurations leading to a very large number of independent variables in the regression (see footnote 11). Secondly, a given characteristics configuration must occur in at least two time periods in order to identify the time-dummy parameters. However, this method offers an interesting avenue for future research in the hedonic literature.

10. Conclusion:

This paper has discussed the performance of the hedonic regression time-dummy method for constructing price indexes with regard to the monotonicity axioms. The paper shows that the index method does not satisfy any of four versions of the monotonicity axioms. This calls into question the use of this method for the construction of price indexes. However, like in the case of the failure of the Tornqvist, Sato-Vartia and Geometric Mean Indexes there may be an economic explanation as to why the

[11] For example, if there are two different quality features and the first can take three different values while the latter can take two different values then there are potentially six different categorisations. In general if there are k variables and each variable has m_k different configurations then we have a maximum of $m_1 \times m_2 \times ... \times m_k$ different configurations.

monotonicity axiom is violated. In the case of the conventional price indexes the reason identified by Reinsdorf and Dorfman (1999) for the failure of monotonicity was that quantities were not treated as endogenous – when prices changed the quantities should also be allowed to change. An analogous economic explanation for the failure of monotonicity of the time-dummy hedonic method is that we allowed the market assessment of the value of characteristics to change when we changed prices. But this is like assuming that the market equilibrium is different in the two cases.

Whether the economic explanation of the failure of monotonicity is as persuasive in the case of hedonics as for the cost-of-living indexes is debatable. The way in which the hedonic method is most often justified is not with reference to economic models of hedonic functions but as essentially a statistical method for decomposing the price of a good into various components. In this context the failure of the monotonicity axiom appears to be unacceptable and it would seem preferable if hedonic methods were nested within alternative index calculation methods.

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12. Appendix:

Table 4: Regression 1 – Pooled Regression.

	Coefficients (std. err.) for the Pooled Regression
	1972 - 84
Dependent Var.	Log(Price)
R-Sq.	0.8383
Adj. R-Sq.	0.8086
F-Stat. **(P-Value)**	28.15 (<0.0001)
No. Observations	91
Intercept	9.4283 (0.7568)
Log(Speed)	0.3909 (0.1264)
Log(Capacity)	0.4588 (0.0792)
D1973	0.0160 (0.1212)
D1974	-0.2177 (0.1151)
D1975	-0.3093 (0.1152)
D1976	-0.4173 (0.1109)
D1977	-0.4167 (0.123)
D1978	-0.5740 (0.1689)
D1979	-0.7689 (0.1689)
D1980	-0.9602 (0.1623)
D1981	-0.9670 (0.1608)
D1982	-0.9537 (0.1695)
D1983	-1.1017 (0.184)
D1984	-1.1812 (0.1887)

Table 5: Regression 2 – Adjacent Period Regressions.

	Coefficients (std. err.) of each Adjacent Period Regression					
	1972 - 73	**1973 - 74**	**1974 - 75**	**1975 - 76**	**1976 - 77**	**1977 - 78**
Dependent Var.	Log(Price)	Log(Price)	Log(Price)	Log(Price)	Log(Price)	Log(Price)
R-Sq.	0.9103	0.7193	0.6693	0.7307	0.7417	0.8294
Adj. R-Sq.	0.8896	0.6257	0.5701	0.6686	0.6900	0.7782
F-Stat. (P-Value)	43.97 (<0.0001)	7.69 (0.0075)	6.75 (0.0091)	11.76 (0.0005)	14.35 (0.0001)	16.20 (0.0004)
No. Observations	17	13	14	17	19	14
Intercept	9.6241 (1.994)	6.2790 (6.6167)	6.0304 (5.3254)	9.1046 (1.6049)	8.26 (1.169)	7.0177 (1.0311)
Log(Speed)	0.5265 (0.3375)	0.2345 (0.7808)	0.2487 (0.6108)	0.4411 (0.2625)	0.1297 (0.2225)	-0.2791 (0.2071)
Log(Capacity)	0.5336 (0.1956)	1.0023 (0.8316)	1.0158 (0.6979)	0.4965 (0.1756)	0.4418 (0.1077)	0.4528 (0.0878)
D1973	-0.0764 (0.1358)	---	---	---	---	---
D1974	---	-0.2306 (0.1743)	---	---	---	---
D1975	---	---	-0.0920 (0.1747)	---	---	---
D1976	---	---	---	-0.1277 (0.1435)	---	---
D1977	---	---	---	---	0.0563 (0.1227)	---
D1978	---	---	---	---	---	-0.0303 (0.1041)

Table 5 (contd.): Regression 2 – Adjacent Period Regressions.

	Coefficients (std. err.) of each Adjacent Period Regression					
	1978 - 79	**1979 - 80**	**1980 - 81**	**1981 - 82**	**1982 - 83**	**1983 - 84**
Dependent Var.	Log(Price)	Log(Price)	Log(Price)	Log(Price)	Log(Price)	Log(Price)
R-Sq.	0.7000	0.8690	0.8592	0.9381	0.9763	0.9099
Adj. R-Sq.	0.5500	0.8129	0.8170	0.9195	0.9674	0.8713
F-Stat. **(P-Value)**	4.67 (0.0520)	15.48 (0.0018)	20.34 (0.0001)	50.47 (<0.0001)	109.98 (<0.0001)	23.57 (0.0005)
No. Observations	10	11	14	14	12	11
Intercept	-0.223 (12.7674)	2.0219 (2.5211)	2.5591 (3.557)	6.3986 (2.9565)	8.8678 (1.6983)	9.0288 (3.6142)
Log(Speed)	-1.5152 (1.5912)	-1.0664 (0.3206)	-0.5108 (0.479)	0.0577 (0.3995)	0.4463 (0.231)	0.4631 (0.5001)
Log(Capacity)	1.0257 (1.2842)	0.8453 (0.2613)	0.981 (0.3463)	0.6344 (0.2853)	0.4219 (0.1644)	0.3839 (0.3507)
D1979	-0.1948 (0.0554)	---	---	---	---	---
D1980	---	-0.1611 (0.0339)	---	---	---	---
D1981	---	---	0.0089 (0.0649)	---	---	---
D1982	---	---	---	0.0214 (0.0546)	---	---
D1983	---	---	---	---	-0.1532 (0.0566)	---
D1984	---	---	---	---	---	-0.077 (0.1067)

Table 6: The Indexes for Hard Disk Drives Compared.

Year	Pooled Regression	Adjacent Period Regression (Chained)
1972	1.00	1.00
1973	1.02	0.93
1974	0.80	0.74
1975	0.73	0.67
1976	0.66	0.59
1977	0.66	0.62
1978	0.56	0.61
1979	0.46	0.50
1980	0.38	0.42
1981	0.38	0.43
1982	0.39	0.44
1983	0.33	0.38
1984	0.31	0.35