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## **Inexact Index Numbers and Economic Monotonicity Violations: The GDP Implicit Price Deflator**

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### **Abstract**

Several countries have recently switched – or are about to do so – to chained price and quantity indexes in the framework for their national accounts. Thus, the United States and Canada have adopted the chained Fisher indexes, whereas the United Kingdom, Switzerland, Australia and New Zealand have opted for chained Laspeyres indexes for real GDP, and chained Paasche for the implicit price deflator. Nonetheless, the vast majority of countries, including most OECD members, have not yet embraced chaining. In these countries the GDP implicit price deflator is still computed as a *direct* (or fixed-base) Paasche price index. Time series data on the deflator are obtained by taking *runs* (or sequences) of these direct indexes. Changes in the price level over consecutive periods are measured by the change in the direct Paasche index, a use for which it is ill suited. Indeed, using the economic approach to index numbers in the context of supply theory, we show that runs of direct Paasche indexes fail an economic monotonicity test if the number of periods exceeds two. That is, the price index can register a drop between consecutive periods even though none of the disaggregate prices has fallen, and some have actually increased. The purpose of this paper is thus to draw attention to some of the undesirable properties of the direct Paasche price index, and, more generally, to the problems that the absence of chaining can raise. Examples based on the *CES/CET* aggregator function are provided. A similar result holds for runs of direct Fisher indexes. This provides a powerful argument in favor of chaining. It should also serve as a warning against the use of unit values. Yet these are routinely used as elementary price indexes at the most disaggregate level.

*Keywords:* Paasche index, Fisher index, chain index numbers, economic monotonicity, real GDP, GDP implicit price deflator

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# **Inexact Index Numbers and Economic Monotonicity Violations: The GDP Implicit Price Deflator**

## **0. Introduction**

Several countries have recently switched – or are about to do so – to chained price and quantity indexes in the framework of their national accounts. Thus, the United States and Canada have adopted the chained Fisher index, whereas the United Kingdom, Switzerland, Australia, and New Zealand, among others, have opted for the chained Laspeyres index as far as real GDP is concerned, and the chained Paasche for the implicit price deflator. Nonetheless, the vast majority of countries, including many OECD members, have not yet embraced chaining. In these countries the GDP implicit price deflator is still computed as a *direct* (or fixed-base) Paasche price index. Time series data on the deflator are obtained by taking *runs* (or sequences) of these direct indexes. Changes in the price level over consecutive periods are measured by the change in the direct Paasche index, a use for which it is unsuited. Indeed, using the economic approach to index numbers in the context of supply theory, one can show that direct Paasche price indexes fail an economic monotonicity test if the number of periods exceeds two.<sup>1</sup> That is, the price index can register a drop between consecutive periods even though none of the disaggregate prices has fallen, and some have actually increased. This makes the direct Paasche price index ill suited for economic analysis in a time-series context. The direct Paasche GDP deflator, in particular, should not be viewed as a good measure of the general price level, and changes in the deflator should not be used as a yardstick of inflation. Similar difficulties, although to a lesser degree, affect the direct Fisher price index. The use of unit values in lieu of elementary price indexes should also be avoided. Our results provide a strong argument in favor of chaining.

Our analysis is based on the economic approach to index numbers. Much of this literature focuses on exact index numbers, to use the terminology introduced by Diewert (1976). That is, knowing the precise form of the aggregator function (e.g. the production possibilities frontier), one seeks to find an index number formula that is exact for it. Alternatively, starting from an arbitrary index number formula, one looks for the aggregator function for which this index number would be

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<sup>1</sup> This problem also arises in the context of demand theory; see Kohli (1986). However, Paasche index numbers are probably most prevalent in supply theory, since they are widely used in the national accounts.

exact.<sup>2</sup> For instance, it turns out that the Paasche price index is exact as long as the transformation function is either linear or Leontief. The strategy followed in this paper is somewhat different. We investigate the properties of a given, commonly used index number formula when we know in advance that this index is *not* exact for the underlying aggregator function, which itself is assumed to be fairly general and well behaved. Specifically, we will look at the direct Paasche price index (the Laspeyres and the Fisher indexes will be briefly examined as well), while assuming that the production possibilities frontier is strictly concave. The case of the *CET* aggregator function can be thought of as an example of a well-behaved production possibilities frontier. Clearly, one could argue that if the aggregator function is *CET*, then it would be a simple matter to use a *CET* price index, which would then be exact. The point, though, is that analysts and commentators often have no choice in this matter, statistical agencies typically supplying Paasche, Laspeyres and Fisher indexes only. In any case, the selection of the *CET* is only meant as an illustration. Any other functional form that allows the production possibilities frontier to be strictly concave would yield similar results.

## 1. Runs of direct Paasche price indexes

The *direct Paasche price index* ( $P_{t,0}^P$ ) makes a direct comparison between the cost of a basket of goods in the current period (period  $t$ ) and the cost of the same basket at base-period (period  $0$ ) prices. Period- $t$  quantities are used for this comparison. Formally, the direct Paasche price index can be defined as follows:

$$(1) \quad P_{t,0}^P \equiv \frac{\sum_i p_{i,t} q_{i,t}}{\sum_i p_{i,0} q_{i,t}},$$

where  $p_{i,t}$  and  $q_{i,t}$  denote the price and the quantity of good  $i$  at time  $t$ .

A *run* of direct Paasche price indexes is then given by the following sequence:

$$(2) \quad 1, \frac{\sum_i p_{i,1} q_{i,1}}{\sum_i p_{i,0} q_{i,1}}, \frac{\sum_i p_{i,2} q_{i,2}}{\sum_i p_{i,0} q_{i,2}}, \dots, \frac{\sum_i p_{i,t-1} q_{i,t-1}}{\sum_i p_{i,0} q_{i,t-1}}, \frac{\sum_i p_{i,t} q_{i,t}}{\sum_i p_{i,0} q_{i,t}},$$

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<sup>2</sup> Much emphasis has been devoted to superlative indexes, i.e. index numbers that are exact for flexible functional forms. A functional form is flexible if it can provide a second-order approximation to an arbitrary aggregator function. The terms “flexible”, “superlative”, and “aggregator function” were all three coined by Diewert (1974, 1976),

or, in more compact form:

$$(3) \quad 1, P_{1,0}^P, P_{2,0}^P, \dots, P_{t-1,0}^P, P_{t,0}^P.$$

It is common practice to use elements of this sequence to make comparisons between arbitrary pairs of periods. For instance, if one wanted to compare period  $t$  with period  $t-1$ , one would calculate  $\Pi_{t,t-1}$  defined as follows:

$$(4) \quad \Pi_{t,t-1} \equiv \frac{P_{t,0}^P}{P_{t-1,0}^P} = \frac{\sum_i p_{i,t} q_{i,t}}{\sum_i p_{i,0} q_{i,t}} \frac{\sum_i p_{i,0} q_{i,t-1}}{\sum_i p_{i,t-1} q_{i,t-1}}.$$

It is important to note that, although  $\Pi_{t,t-1}$  rests on the comparison between two Paasche indexes, it is not itself a Paasche index, unless period  $t-1$  happens to be the base period.<sup>3</sup> Although its properties are little known,  $\Pi_{t,t-1}$  is routinely used in economic analysis. Consecutive changes in the GDP price deflator, in particular, are often used as a broad measure of inflation. If  $\Pi_{t,t-1}$  turns out to be greater than one, one might be inclined to conclude that, on average, prices have gone up between period  $t-1$  and period  $t$ . As we shall see below, this conclusion could be diametrically wrong.

## 2. Profit maximization and the economic approach to index numbers

The economic approach to index numbers assumes that the observed quantities ( $q_{i,t}$ ) are not random, but rather that they are the outcome of economic decisions. In particular, they reflect prices, technology, and optimizing behavior. Assume for simplicity that national production involves just one input (e.g. an aggregate of labor and capital), the quantity of which we denote by  $x_t$ , and two outputs ( $i = 1, 2$ ). We assume constant returns to scale, nondecreasing marginal rates of transformation, and profit maximization. Let the country's production possibilities frontier be given by the following transformation (or factor requirements) function:

$$(5) \quad x_t = h(q_{1,t}, q_{2,t}).$$

Constant returns to scale imply that  $h(\cdot)$  is linearly homogeneous, and the assumption of nondecreasing marginal rates of transformation means that  $h(\cdot)$  is convex. Profit maximization implies that the marginal rate of transformation is equal to the output price ratio:

$$(6) \quad \frac{h_1(q_{1,t}, q_{2,t})}{h_2(q_{1,t}, q_{2,t})} = \frac{p_{1,t}}{p_{2,t}},$$

where  $h_i(\cdot) \equiv \partial h(\cdot) / \partial q_{i,t}$ ,  $i = 1, 2$ . It is possible to solve (5) and (6) for  $q_{i,t}$ :

$$(7) \quad q_{i,t} = q_i(p_{1,t}, p_{2,t}, x_t) = y_i(p_{1,t}, p_{2,t}) x_t,$$

where  $y_i(\cdot) \equiv q_i(\cdot) / x_t$  can be interpreted as unit output (output per unit of input). The linear homogeneity of  $q_i(\cdot)$  with respect to  $x_t$  results from the linear homogeneity of  $h(\cdot)$ . Moreover, the assumption of optimization and the convexity of  $h(\cdot)$  (concavity of the production possibilities frontier) imply that  $q_i(\cdot)$  – and  $y_i(\cdot)$  for that matter – is homogeneous of degree zero in prices and nondecreasing in its own price:

$$(8) \quad \frac{\partial q_i(\cdot)}{\partial p_{1,t}} p_{1,t} + \frac{\partial q_i(\cdot)}{\partial p_{2,t}} p_{2,t} = 0, \quad \frac{\partial q_i(\cdot)}{\partial p_{i,t}} \geq 0, \quad i = 1, 2.$$

The economic approach to index numbers then amounts to introducing expressions such as (7) into index number formulas such as (1). In the two-good case, the direct Paasche price index therefore becomes:

$$(9) \quad P_{t,0}^P = \frac{p_{1,t} q_1(p_{1,t}, p_{2,t}, x_t) + p_{2,t} q_2(p_{1,t}, p_{2,t}, x_t)}{p_{1,0} q_1(p_{1,t}, p_{2,t}, x_t) + p_{2,0} q_2(p_{1,t}, p_{2,t}, x_t)} = \frac{p_{1,t} y_1(p_{1,t}, p_{2,t}) + p_{2,t} y_2(p_{1,t}, p_{2,t})}{p_{1,0} y_1(p_{1,t}, p_{2,t}) + p_{2,0} y_2(p_{1,t}, p_{2,t})}.$$

It is customary to normalize base-period (period 0) prices to unity ( $p_{1,0} = p_{2,0} = 1$ ).

Expression (9) can therefore be rewritten as:

$$(10) \quad P_{t,0}^P = P_{t,0}^P(p_{1,t}, p_{2,t}) = \frac{p_{1,t} y_1(p_{1,t}, p_{2,t}) + p_{2,t} y_2(p_{1,t}, p_{2,t})}{y_1(p_{1,t}, p_{2,t}) + y_2(p_{1,t}, p_{2,t})}.$$

### 3. Economic Monotonicity Test

It seems reasonable to expect a well-behaved measure of the general price level to be monotonically increasing – or at least nondecreasing – in its arguments.<sup>4</sup> That is, if one disaggregate price were to rise, while all other prices are held constant, one would like to see the aggregate price index increase, or at least not fall, *after having allowed for the endogenous*

<sup>3</sup> This is also pointed out by Afriat (1977).

<sup>4</sup> See Kohli (1986) for an examination of the monotonicity properties of index numbers in the context of demand theory.

*adjustment in quantities.* To investigate the slope properties of the direct Paasche price index, it suffices to differentiate (10) with respect to a disaggregate price, say the first one:

$$\begin{aligned}
\frac{\partial P_{t,0}^P}{\partial p_{1,t}} &= \frac{\left( y_{1,t} + p_{1,t} \frac{\partial y_{1,t}}{\partial p_{1,t}} + p_{2,t} \frac{\partial y_{2,t}}{\partial p_{1,t}} \right) (y_{1,t} + y_{2,t}) - (p_{1,t} y_{1,t} + p_{2,t} y_{2,t}) \left( \frac{\partial y_{1,t}}{\partial p_{1,t}} + \frac{\partial y_{2,t}}{\partial p_{1,t}} \right)}{(y_{1,t} + y_{2,t})^2} \\
&= \frac{y_{1,t}^2 + y_{1,t} y_{2,t} + y_{2,t} \frac{\partial y_{1,t}}{\partial p_{1,t}} (p_{1,t} - p_{2,t}) + y_{1,t} \frac{\partial y_{2,t}}{\partial p_{1,t}} (p_{2,t} - p_{1,t})}{(y_{1,t} + y_{2,t})^2} \\
(11) \quad &= \frac{y_{1,t}^2 + y_{1,t} y_{2,t} + y_{1,t} y_{2,t} (\varepsilon_{11,t} - \varepsilon_{21,t}) \left( 1 - \frac{p_{2,t}}{p_{1,t}} \right)}{(y_{1,t} + y_{2,t})^2} \\
&= \frac{y_{1,t}^2 + y_{1,t} y_{2,t} \left[ 1 - \theta_{12,t} \left( 1 - \frac{p_{2,t}}{p_{1,t}} \right) \right]}{(y_{1,t} + y_{2,t})^2} \begin{matrix} \geq \\ < \end{matrix} 0
\end{aligned}$$

where  $\varepsilon_{11,t} \equiv \partial \ln y_{1,t} / \partial \ln p_{1,t} \geq 0$  and  $\varepsilon_{21,t} \equiv \partial \ln y_{2,t} / \partial \ln p_{1,t} \leq 0$  are the price elasticities of output supply with respect to  $p_{1,t}$  at time  $t$ ;  $\theta_{12,t} \equiv [h(\cdot)h_{12}(\cdot)/h_1(\cdot)h_2(\cdot)] \leq 0$ , where  $h_{12}(\cdot) \equiv \partial^2 h(\cdot) / (\partial q_{1,t} \partial q_{2,t})$ , is the elasticity of transformation between the two outputs. Note that  $\varepsilon_{11,t} = -\theta_{12,t} s_{2,t}$  and  $\varepsilon_{21,t} = \theta_{12,t} s_{1,t}$ ,  $s_{i,t}$  being the revenue share of output  $i$  at time  $t$ ,<sup>5</sup> so that  $\varepsilon_{11,t} - \varepsilon_{21,t} = -\theta_{12,t} \geq 0$ .<sup>6</sup> It is apparent from the last line of (11) that  $\partial P_{t,0}^P / \partial p_{1,t}$  can be negative if the ratio  $p_{2,t}/p_{1,t}$  is sufficiently large, i.e. if  $p_{1,t}$  is small enough relative to  $p_{2,t}$ , unless  $\theta_{12,t}$  happens to be nil.<sup>7</sup> If the production possibilities frontier is strictly concave, on the other hand, and if production is diversified,  $\theta_{12,t}$  is strictly negative. In that case, which can be viewed as the normal case, the Paasche price index (10) fails to be globally monotonically increasing in its components. That is, it may register a fall as  $p_{1,t}$  increases and  $p_{2,t}$  is held constant. Since the price ratio is unity in the base period, economic monotonicity violations can only occur at a point away from the base period; that is, if one compares two situations that do not encompass the base period.

<sup>5</sup> See Kohli (1991), for instance.

<sup>6</sup> Note that it follows from (8) that  $\varepsilon_{11,t} \geq 0$  and  $\varepsilon_{21,t} = -\varepsilon_{22,t} \leq 0$ .

<sup>7</sup> This would be the case if the factor requirements function were linear or Leontief, in which case the Paasche price index would be exact and thus necessarily well behaved.

#### 4. Numerical illustration

A simple numerical example might help to show how economic monotonicity violations might occur. Assume three periods, periods 0, 1, and 2. Table 1 shows the given values of  $p_1$  and  $p_2$  for the three periods, together with the optimizing values of  $q_1$  and  $q_2$ , assuming that the factor requirements function is given by  $z = q_{1,t}^2 + q_{2,t}^2$ , a quarter circle in the first quadrant.<sup>8</sup>

**Table 1**  
**Economic monotonicity violations: Numerical illustration**

$t$	$p_{1,t}$	$p_{2,t}$	$q_{1,t}$	$q_{2,t}$	$P_{t,0}^P$
0	1.0000	1.0000	1.0000	1.0000	1.0000
1	0.2000	1.0000	0.2774	1.3868	0.8667
2	0.4000	1.0000	0.5252	1.3131	0.8286

It is then straightforward to compute the direct Paasche price index. It is shown in the last column, and it can be seen that between periods 0 and 1, as  $p_1$  falls from 1 to 0.2 while  $p_2$  remains unchanged,  $P^P$  drops from 1 to 0.8667. Between periods 1 and 2,  $p_1$  recovers and increases from 0.2 to 0.4, while  $p_2$  is still being held constant, but  $P^P$  keeps dropping, from 0.8667 to 0.8286. That is, the GDP price deflator registers a fall, thus suggesting deflation (at a rate of 4.4%), even though one disaggregate price has doubled and the other one has remained unchanged.

#### 5. The CET aggregator function

The behavior of the direct Paasche price index can be further investigated with the help of some simulations. Let the country's factor requirements function (5) have the following CET form:<sup>9</sup>

$$(12) \quad x_t = (q_{1,t}^\rho + q_{2,t}^\rho)^{1/\rho}, \quad \rho > 1 .$$

<sup>8</sup> The supply of output  $i$  is then given by  $q_{i,t} = p_{i,t} \sqrt{2/(p_{1,t}^2 + p_{2,t}^2)}$ ; see expression (13) below.

<sup>9</sup> The name CET stands for *constant elasticity of transformation*. The elasticity of transformation implied by (12) can be calculated as  $\theta_{12} = 1/(1 - \rho)$ .

Under profit maximization, the output supply functions are:

$$(13) \quad q_{i,t} = \frac{p_{i,t}^{1/(\rho-1)}}{\left[ p_{1,t}^{\rho/(\rho-1)} + p_{2,t}^{\rho/(\rho-1)} \right]^{1/\rho}} x_t, \quad i = 1, 2.$$

The GDP function can then be written as:<sup>10</sup>

$$(14) \quad \pi(p_{1,t}, p_{2,t}, x_t) = \left[ p_{1,t}^{\rho/(\rho-1)} + p_{2,t}^{\rho/(\rho-1)} \right]^{(\rho-1)/\rho} x_t = r(p_{1,t}, p_{2,t}) x_t,$$

where

$$(15) \quad r(p_{1,t}, p_{2,t}) \equiv \left[ p_{1,t}^{\rho/(\rho-1)} + p_{2,t}^{\rho/(\rho-1)} \right]^{(\rho-1)/\rho}$$

is the unit revenue function. The following *CET* price index will then be *exact* in the sense of Diewert (1976):

$$(16) \quad P_{t,0}(p_{1,t}, p_{2,t}) = \frac{r(p_{1,t}, p_{2,t})}{r(p_{1,0}, p_{2,0})} = \frac{\left[ p_{1,t}^{\rho/(\rho-1)} + p_{2,t}^{\rho/(\rho-1)} \right]^{(\rho-1)/\rho}}{2^{(\rho-1)/\rho}}.$$

Given that  $p_{1,0} = p_{2,0} = 1$ , it follows from (13) that  $q_{1,0} = q_{2,0}$ . The direct Laspeyres price index can thus be written as:

$$(17) \quad P_{t,0}^L(p_{1,t}, p_{2,t}) \equiv \frac{p_{1,t} q_{1,0} + p_{2,t} q_{2,0}}{q_{1,0} + q_{2,0}} = \frac{1}{2} (p_{1,t} + p_{2,t}).$$

It is obvious from (17) that the direct Laspeyres price index is monotonically increasing in prices. Next, making use of (13), we can derive the direct Paasche price index. It is as follows:

$$(18) \quad P_{t,0}^P(p_{1,t}, p_{2,t}) \equiv \frac{p_{1,t} q_{1,t} + p_{2,t} q_{2,t}}{q_{1,t} + q_{2,t}} = \frac{p_{1,t}^{\rho/(\rho-1)} + p_{2,t}^{\rho/(\rho-1)}}{p_{1,t}^{1/(\rho-1)} + p_{2,t}^{1/(\rho-1)}}.$$

As indicated by (11), and given that  $\theta_{12}$  is strictly negative in the *CET* case, the direct Paasche price index (18) is not globally monotonically increasing in prices.

The direct Fisher index, finally, can be obtained as:

$$(19) \quad P_{t,0}^F(p_{1,t}, p_{2,t}) \equiv \sqrt{P_{t,0}^L(p_{1,t}, p_{2,t}) P_{t,0}^P(p_{1,t}, p_{2,t})} = \sqrt{\frac{1}{2} (p_{1,t} + p_{2,t}) \frac{p_{1,t}^{\rho/(\rho-1)} + p_{2,t}^{\rho/(\rho-1)}}{p_{1,t}^{1/(\rho-1)} + p_{2,t}^{1/(\rho-1)}}}.$$

Note that when  $\rho = 2$ ,  $P_{t,0}^F(p_{1,t}, p_{2,t}) = P_{t,0}(p_{1,t}, p_{2,t})$ , i.e. the direct Fisher index is exact for aggregator function (12). This is because in that case the *CET* factor requirements function

<sup>10</sup> See Kohli (1978, 1991) and Woodland (1982) for details.

becomes a special case of a quadratic mean of order 2 (the square rooted quadratic function), and, as shown by Diewert (1976), the Fisher index is exact for that functional form. In this case, the direct Fisher price index is necessarily monotonically increasing in prices. However, for other values of  $\rho$  this may no longer be true, since it might well be that the adverse behavior of the Paasche component dominates that of its Laspeyres counterpart.

Our results can easily be illustrated with the help of some simulations. We show in Figure 1  $P_{t,0}^P(p_1, p_2)$  as a function of  $p_1$  for alternative values of  $\rho$ , after having set  $p_2 = 1$ . It is apparent that  $P_{t,0}^P(p_1, p_2)$  is not monotonically increasing in  $p_1$  for low values of  $p_1$ . That is, as  $p_1$  increases, the GDP price deflator will actually fall, even though  $p_2$  is held constant by assumption.

Next, in Figure 2, we show  $P_{t,0}^P(p_1, p_2)$ ,  $P_{t,0}^F(p_1, p_2)$ , and  $P_{t,0}^L(p_1, p_2)$  as functions of  $p_1$  for  $\rho = 4$ . It can be seen that, although  $P_{t,0}^P(p_1, p_2) \geq P_{t,0}^F(p_1, p_2) \geq P_{t,0}^L(p_1, p_2)$  as expected, there are ranges for  $p_1$  where  $\partial P_{t,0}^P(p_1, p_2) / \partial p_1 < \partial P_{t,0}^F(p_1, p_2) / \partial p_1 < \partial P_{t,0}^L(p_1, p_2) / \partial p_1$ , and where, moreover,  $P_{t,0}^P(p_1, p_2)$  and  $P_{t,0}^F(p_1, p_2)$  are decreasing in  $p_1$ .

Finally, it can be seen from Figures 1 and 2 that for  $p_1 > 0$ ,  $P_{t,0}^P(p_1, 1) \geq P_{t,0}^P(1, 1)$  if and only if  $p_1 \geq 1$ . The same is true for the direct Fisher index. That is, *relative to the base period*, both indexes are monotonically increasing in prices. This provides a strong argument in favor of chaining.

## 6. Economic monotonicity violations in the primal space

The fact that the direct Paasche index fails the economic monotonicity test can also be seen in the primal quantity space. Figure 3 shows the production possibilities frontier in the  $(q_1, q_2)$  space. Assume that  $p_1 = p_2 = 1$  to start. The initial equilibrium is at point **A**. Nominal GDP is given by the distance **OB**. The Laspeyres quantity index (which is equal to  $q_1 + q_2$ ) is given by **OB** as well, which indicates that the Paasche price index is equal to unity. Assume now that  $p_1$  is falling, while  $p_2$  remains constant. The new price ratio is given by the slope of **EE'**, and equilibrium moves to **C**. Nominal GDP is now equal to the distance **OE**, whereas the Laspeyres quantity index is equal to **OD**. The Paasche price index is now given by the ratio **OE/OD**, which is clearly less than unity. If we let  $p_1$  fall further, still holding  $p_2$  constant, equilibrium will tend towards **F**. It can be seen that

at  $F$  nominal GDP and the Laspeyres quantity index are both equal to the distance  $OF$ , which implies that the Paasche price index is again equal to one. That is, moving from  $C$  to  $F$ , the Paasche price index has increased, even though  $p_2$  was held constant, and  $p_1$  has fallen. Note that this result is valid for any well-behaved, concave production possibilities frontier, not just for the  $CET$  functional form.

## 7. Unit values

Economic monotonicity violations are likely to plague unit values even more so than they do direct Paasche indexes. Yet unit values are routinely used at the elementary level to aggregate goods seemingly belonging to a same category. The average price of an apparently homogeneous product – yet sold at different times, different places, under different conditions – is often calculated by simply dividing total revenues by the number of items sold. The use of unit values is particularly prevalent in the area of foreign trade, where differences in quality and specification are not always taken into account when calculating the average price of exported cars, cameras, or watches. Thus, import and export price deflators are often computed on the basis of unit values. Unit values can be thought of as a special (and rather perverse) case of direct Paasche price indexes, for it is defined relative to an arbitrary (and often imaginary) base period. The unit value index can be defined as follows:

$$(20) \quad P_t^U(p_{1,t}, p_{2,t}) \equiv \frac{p_{1,t}q_{1,t} + p_{2,t}q_{2,t}}{q_{1,t} + q_{2,t}} .$$

Comparing (20) with (18), it can be seen that  $P_t^U$  is equivalent to  $P_{t,0}^P$  if we interpret  $P_t^U$  as being defined relative to a base period for which  $p_{1,0} = p_{2,0}$ . Naturally, when we aggregate different types of goods, such as luxury automobiles and compact cars, there may well never have been such a time. Since direct indexes only make sense relative to the base period, one must conclude that they are meaningless if that base period has never existed. Economic monotonicity violations are likely to be particularly important in the case of unit values since relative prices will tend to differ greatly from unity. As an example, high performance automobiles can easily cost 20 times as much as compact cars, top-of-the-line cameras can cost 50 times more than more basic ones, luxury watches can be worth 1,000 times more than low-end ones, and so on.

## 8. Conclusion

It is well known that, in the context of supply theory, the Paasche price index tends to lie above the true price level, and the Laspeyres index underneath it. This is apparent in Figure 2 where  $P_{t,0}^P$  always lies above  $P_{t,0}^L$ . This is often understood to mean that the Paasche price index overstates price increases, and that the Laspeyres index underestimates them. This is clearly does not need to be true if the reference period is not the base period, i.e. the period for which the data are normalized. Thus, it is obvious from Figure 2 that  $P_{t,0}^P$  can fall at the same time as  $P_{t,0}^L$  is increasing. Note also that relative to the base period, the direct Paasche price index will understate price falls, whereas as the Laspeyres index will exaggerate them. At a time when several countries have been flirting with deflation, this might be something to keep in mind.

The fact that the direct Paasche GDP deflator is not monotonically increasing in prices makes it a poor indicator of inflation, since it might point at a price increase when prices are actually falling, and vice-versa. Yet it is widely used in the literature, including in the areas of monetary economics and macroeconomics. For instance, Taylor (1993) has used it as a measure of inflation in his famous rule for monetary policy, and this well before chaining was introduced in the United States.

Naturally, the criticism applying to the GDP deflator can also be addressed to the other implicit price deflators in the context of the national accounts. These deflators typically have the direct Paasche form as well, and thus they fail to be globally monotonically increasing in their price components. Yet, some of these indexes are closely scrutinized. The deflator of consumption expenditures, for instance, is often used as a yardstick of inflation. In view of its failure of the economic monotonicity test, this would seem rather inappropriate.

There are other reasons why the use of the direct Paasche GDP deflator as a measure of the price level should be avoided, independently of whether chaining takes place or not. Thus, the Paasche functional form is unduly restrictive. As already noted, it is exact for very restrictive aggregator functions only (Linear and Leontief). Superlative indexes, which are exact for flexible aggregator functions, are therefore to be preferred. Second, GDP price deflators incorporate terms-of-trade changes, which are fundamentally a real – not a price – phenomenon (this point also applies when the GDP deflator is measured by a superlative index and when it is chained). The problem with the standard procedure becomes apparent if import prices fall, for instance. This will

increase the GDP price deflator (since import prices enter the calculation of the GDP deflator with a negative weight), even though this shock is clearly not inflationary, quite the contrary.<sup>11</sup>

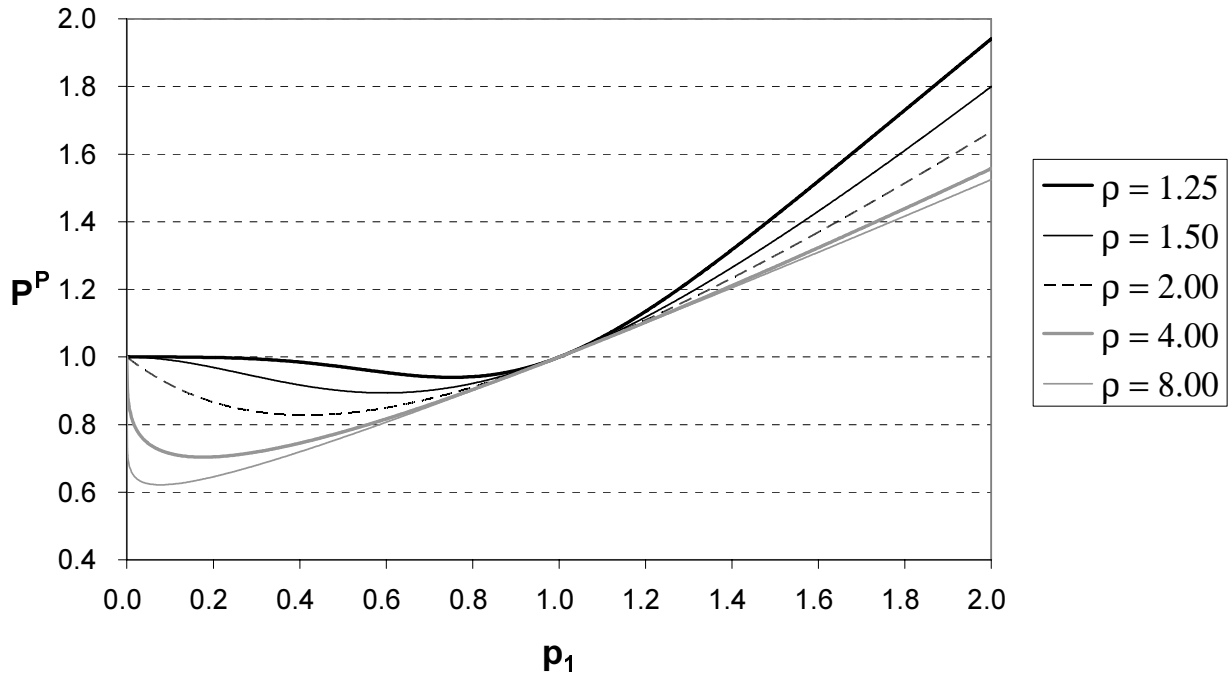
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<sup>11</sup> See Kohli (2004) for additional details.

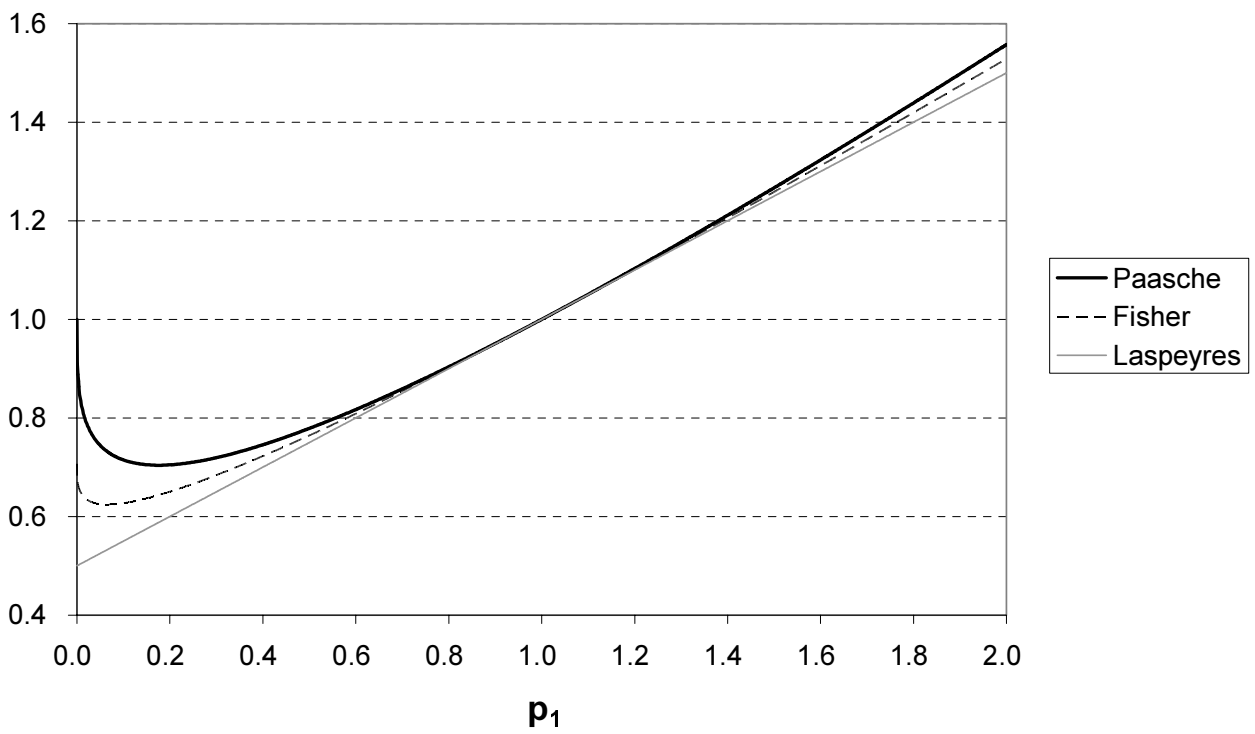
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**Figure 1**  
**Direct Paasche price index for alternative values of  $\rho$**



**Figure 2**  
**Direct Paasche, Fisher and Laspeyres indexes for  $\rho = 4$**



**Figure 3**  
**The production possibilities frontier and the GDP implicit price deflator**

