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Obsolescence vs. Deterioration with Embodied Technological Change

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Abstract

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Few issues in economics have been as contentious longer than the role of capital in production and distribution. Das Kapital still resonates for some and raises body temperatures of others. The Marxist-capitalist conflict flared up in the Two Cambridge's debate in the 1950s. A less ideological version resurfaced in the Denison vs. Jorgenson and Griliches debate in the 1970s. Today a low-grade rumble persists over how to measure the capital input. This paper takes the neoclassical approach to address a central aspect of today's debate. When technological change is embodied in new capital, should statistical measures of capital distinguish between obsolescence of old capital and deterioration? I argue that this question underlies concerns by many about the use of depreciation estimates in measuring net capital stocks.

I make the case here that no distinction should be made between obsolescence and deterioration in constructing capital measures. I also argue that the ratios of user-costs equal relative efficiencies that ordinarily one should not adjust for variations in utilization. I make two arguments. One, a coherent measure of the capital input requires aggregation in comparable units and this requires that adjustments be made both for deterioration and obsolescence. Two, rational decisions by capital users are independent of the obsolescence-deterioration distinction. The first half of the paper deals directly with capital measurement under embodied technical change. The second with behavior, in response to new technological innovations, of a representative rational producer who is using old capital.

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Introduction

Several stubborn questions continue to plague the statistical agencies that produce estimates of the capital input. How does one model capital when technological change is embodied in new vintages of capital? In measuring capital stocks, does one adjust for obsolescence as well as deterioration? If so, how does one do so? Is the effect of obsolescence on capital use different from the effect of deterioration? What role does utilization have to do with capital measures? These issues tend to deal with measurement, but I am also interested in how economic agents respond when their used assets become obsolete, because economic measurement should reflect economic activity.

These questions have two interrelated aspects to them. One question deals with directly with the measurement of the capital input. How does one aggregate capital when technological change renders old assets obsolete relative to new vintages? This question is important to statistical agencies charged with measuring capital, productivity and other aspects of real, as opposed to financial, aggregate economic activity. The other question deals with how economic agents respond to changes in price resulting from obsolescence vs. changes resulting from deterioration. If a new "better" asset model enters the market, how do users of old models respond? Answers to the behavioral questions will have implications for the measurement questions. The context though of my interest in behavior is with issues of aggregate capital measurement.

The issues raised here are by no means new. In fact, disputes about physical capital have persisted many years often flaring up into contentious debates. Today interest among economists has been limited to the measurement community margin. Historical disputes about capital have been ideological, theoretical and statistical. To avoid confusion in a field rife with confusion, I first put this paper in context with respect to relatively recent historical treatment of physical capital.

Next, in the same spirit, I clarify my use of terms like depreciation, obsolescence, deterioration, age, and vintage. These terms have been used by different authors to mean different things. Much confusion can be avoided if we are clear about exactly what we mean when we use these terms. I will use these terms as they have been used in the classical economics literature on depreciation from Hotelling (1925) to Jorgenson (1974) as well as from recent papers on the subject by Wykoff (1970, 2003), Oliner (1993, 1996) and Hulten and Wykoff (1975, 1979, 1981). These definitions are also consistent with definitions by the U.S. Internal Revenue Service for tax purposes.

I then introduce embodied technological change into the famous Jorgenson model of capital. Hulten (1990) discusses several models including one like this. Fourth, I suggest several capital measurement strategies when technological change is embodied in new capital. I will show the assumptions that underlie different measures as well as the conceptual implications of different measures. This paves the way for analysis of producer behavior in response to changes in capital costs resulting from obsolescence compared to deterioration. This analysis buttresses the author's view that obsolescence and deterioration should receive the same treatment in measuring the capital input. It also leads to some observations about the role of utilization of capital. Finally, I sum up.

Overview of the capital controversy

Few issues in economics have been contentious longer than the role of capital. Karl Marx's (1867) Das Kapital still resonates with those who find returns to capital and other aspects of capitalism objectionable. The issues most famous flare up among academic economists in the west was the famous Two Cambridge's debate in the 1950s. The principal protagonists, Joan Robinson for Cambridge England and Robert Solow for

Cambridge, Massachusetts, fired salvos across the Atlantic.[1] Robinson argued that marginal analysis of capital is circular, vacuous and misleading. Solow defended marginal analysis of capital as coherent, substantive and useful. The conflict was energized in part by ideological differences among Marxists and capitalists. A less exotic but important question in the debate was: How to measure capital? Not surprisingly, little was resolved. Each side tired of the argument, and, as Keynesian economics turned focus away from stocks toward flows, researchers on both sides simply went their own ways.

Disagreement on capital stock measurement issues continued to simmer beneath the surface. It flared up again in the 1970s when Dale Jorgenson and Zvi Griliches questioned U.S. Statistical Agencies, whose measures were defended by Edward Denison, on their measures of the capital input in the national accounts.[2] As before, the combatants tired with neither side convincing the other. Since then a low-grade cold war has been going on within the statistical agencies and among academics over many of the same measurement issues. One bone of contention has been whether statistical agencies should measure gross capital stocks or net capital stocks? A related question: Is there one correct capital measure or do different uses of capital data require different measures? The statistical agencies have settled on producing both gross and net capital stock estimates.

Today, even though capital measurement has become esoteric and evidently interests only the measurement community, concern with productivity growth makes capital measurement important to public policy. Hulten in his lucid (1990) paper raises many of the concerns confronting capital measurement statisticians. Here I focus on one aspect of the capital measurement question: When technological change in embodied in new capital, should statistical measures of the capital stock distinguish between obsolescence of old capital and deterioration? I suspect that this question underlies a good deal of the

[1] Several key articles by Robinson and by Solow appear in the 1971 book of readings on growth edited by Harcourt and Laing.
[2] The debate is in the May 1972 Survey of Current Business, US Commerce Department. See Jorgenson, et al. (1974).

current academic disputes involving the treatment of depreciation estimates in measuring the capital input.

Definitions and models

In this paper I define economic depreciation as the decline in price of a cohort of assets with age given date. The phrase economic depreciation is often used in other ways; for example, it may refer to the amount of money set aside to prepare for replacement as an asset wears out, but here we use the phrase to refer only to the decline in price resulting from aging but not the passage of time. The latter is revaluation. As defined here, economic depreciation has two components, because as assets age two distinct types of effects influence price: vintage effects and aging effects.

Deterioration

Deterioration is a relatively straight forward concept. It refers to the decline in the cohort of assets' prices as they age resulting from wear and tear. In principle, deterioration is independent of the specific date. It results from assets getting older and "less efficient". This decline in efficiency may reflect less accuracy, slower performance, increased operating costs, longer down-times, increased frequency of breakdowns and so forth. All of these mean that older assets differ physically (are usually worse) from when they were newer and this is reflected in a change in price. (We are ignoring here start-up costs and learning curves.) In other words, the decline in price attributed to deterioration reflects the decline in physical prowess of the assets—it is an effect that reflects changes in the used asset itself.

The Jorgenson model of economic depreciation

Suppose we use the term decay to refer to the physical process that an asset endures as it is used in production. To model asset decay, Jorgenson (1977) introduced the concept of the efficiency function. At any point in time, the efficiency function is the in-use productive efficiency of assets by age relative to a new asset. Jorgenson normalizes the efficiency of a new asset on one. He then defines the efficiency sequence as the efficiency of assets by age relative to new assets. If we let φ(s) be the efficiency of an age-s asset relative to that of a new asset, then

(1) $\varphi(0) = 1$ and $\varphi(s+1) < \varphi(s)$ for all s = 0, 1, 2, …

Jorgenson links differences in efficiency φ between ages to a mortality sequence, m(s), as:

(2) $m(s) = \varphi(s) - \varphi(s+1)$ for s=0, 1, 2, …,

As the term suggests the mortality sequence indicates the rate of decay in the asset as it works through its life. If the asset has a finite life then the s-sequence terminates at that life, say S. Jorgenson illustrates the efficiency and mortality sequences with several familiar functions: one-horse shay, linear, and geometric.

Jorgenson applies the "duality condition" to in-use assets by equating the cost of using each asset relative to a new one to the efficiency function:

(3) $c(s)/c(0) = \varphi(s)$.

Where c(s) is the user-cost of an asset age-s. Equation (3) is central to the Jorgenson framework because it links the price side of capital to the quantity side. Exploiting this relationship one can link physical decay φ(s) to deterioration, the decline in asset price, q(s), with age. Note that the model distinguishes asset price q from user cost c. They too are linked in the model. In this case, the equation, referred to by Jorgenson as the fundamental price equation of capital theory, is

(4) $$q(0,t) = \sum_{s=0}^{S} \frac{c(s,t+s)}{(1+r)^s}.$$

Equation (4) states that the acquisition price of a new asset in period-t equals the present discounted value of the future stream of user-costs. If the efficiency function φ(s) is stationary then equation (3) allows one to write equation (4) as,

$$(5) \qquad q(0,t) = c(0,t)\sum_{s=0}^{S} \frac{\varphi(s)}{(1+r)^{s}}$$

The duality relationship, equation (3), linking quantities to prices implies that the efficiency function enters both quantity and price measures.

Several important assumptions are embedded in the Jorgenson model. These assumptions may be strong but they are useful, because they make the model tractable and coherent. First the decay rate is a function of time. Clearly this need not be the case, because the decay rate may reflect usage which may differ over time. This is not actually a major problem for the model. It means that the unit of time need not be age in the sense of calendar time but hours of work completed. The model is the same; just the φ-function depends on a different metric.

A second assumption embedded in the Jorgenson model is that the efficiency sequence is stationary over time: φ(s) for all s is independent of date. This assumption while greatly simplifying the model is inessential. Instead of equation (3) we could write:

(6) c(s, t)/c(0, t) = φ(s, t).

In this case φ(s, t) could differ from φ(s, t-1) and from φ(s, t+1). The difficulty with this version of the model is that to use the model to construct capital stocks one needs to identify assets by specific vintage and not simply age. Statistical agencies would have to compile vintage accounts, a rather costly endeavor because it requires perpetual updating of the φ-sequences. Just as φ-functions are easy to deal with, stationary φ-functions are even easier to deal with.

A third more serious assumption embedded in the efficiency function model is that the efficiency of the assets is built into them from the start and is independent of subsequent user choice and independent of the use of other inputs. This would appear to be a very strong assumption that is inconsistent with producers making decisions about frequency of repairs, replacement, scrap, retrofitting, and other decisions that producers surely make. Why does one need this assumption and is it worth the cost in realism? As we noted earlier, the unit of efficiency need not be age but could be hours of use. This transformation, while requiring significantly different and hard to get data, in principle would allow the producer to choose how much to use an asset.

Another implication of built-in efficiencies that satisfy equation (3), however, is harder to avoid. The marginal rate of substitution between age-s and new assets is independent of the use of other inputs. This means that $\varphi(s)$ will not be influenced by changes in the price of labor or of labor efficiency. This is the same assumption required to construct a capital aggregate for use in a production function. When can the function,

$$(7) \qquad Q = F(K1, K2, L)$$

sensibly be written as:

$$(8) \qquad Q = F[K(K1, K2), L] \ ?$$

That is, when is the K-function independent of the labor input? The answer is that the marginal rate of substitution that determines the weights for K1 and K2 must be independent of the marginal product of the labor input.

According to equation (3), the ratio of the user-costs of used to new assets equals the marginal rate of substitution between age-s and new assets. The $\varphi(s)$ term is the marginal rate of substitution between an age-s and a new asset. Assuming φ the marginal rate of substitution between age-s and new assets is a constant is equivalent to the assumption needed to obtain a capital aggregate. Of course, this assumption may be wrong, but then a coherent measure of capital aggregation independent of other aspects of the production process does not exist. In this case, measurement is more complex and appropriate uses of measures in productivity research or policy analysis can be more difficult.

Obsolescence

The concept of obsolescence is a little more subtle than the concept of deterioration. Obsolescence, as defined here, is the effect on the price of a cohort of assets as they age resulting from the availability of new vintages of the assets that have superior characteristics to the older vintage. Nothing actually happens to the used asset, as it did with deterioration. In principle it is unchanged. A newer one is now available, so the older one is relatively obsolete. Obsolescence is a date specific concept—it reflects unique date-specific characteristics of assets. A 1993 Bordeaux wine is a unique vintage of wine defined by the date its grapes were grown. We use the term vintage to mean exactly that—the date the asset was first designed and put into production. With wines, of course, new vintages may be better or worse than previous vintages. The same may be true of other types of capital as well, but seems less likely in a well functioning market economy. If we run out of quality materials and build new assets that are "not as good," then like wine the older vintages of an asset may be superior to newer ones. We've all heard the expression, "they don't make [item] like they used to." Also, classic designs can cause older vintages of certain assets to be extremely valuable—the 1955 Ford Thunderbird for example. Still in modern economies newer assets frequently have features that make them better than older versions

Still for a great many assets, possibly most, the concept of vintage is not as simple as with wine. One cannot, by definition, produce a 1993-vintage wine in 1995; however, with most assets this is not the case. Just because a new design or new feature is available, one does not have to build all new assets with this newly available feature. Not all automobiles, for instance, produced in 2002 had all the newest characteristics available in 2002.

Instead, new features tend to enter the automobile market gradually over time in the sense that very few new cars have the newest features—usually only the top of the line models have the newest features. If a new feature is successful then it is gradually built into lower line models over the years. As Triplett explains it is this tendency of physical capital markets to produce assets containing both new features and old that permits researchers to use hedonic methods to isolate the prices of “quality improvements” from inter-temporal changes in the price of a homogeneous product.

Embodied Technological Change in the Jorgenson Model

We can allow for embodied technological change in the Jorgenson model by letting the efficiency function be vintage specific. The cost of this adaptation from a measurement perspective will be that capital aggregation will involve vintage accounts, a much more complex approach than the usual perpetual inventory method frequently employed. Equation (1) still reflects the efficiency function at any point in time. However, now the efficiency of a new asset, and thus subsequent used assets, will be allowed to change over time as a result of technological change. Suppose embodied technological change occurs at a constant rate θ, then if we normalize on period v = 0, we have

(9) $$\varphi(0,v) = (1 + \theta)^v$$

For example if θ = .01, then the sequence of new asset productive efficiencies will be,

(10) $$1, (1 + .01), (1 + .01)^2, (1 + .01)^3, \ldots$$

We can extend the sequence in equation (10) back in time as well: $(1 + .01)^{-1}$ and so on.

Suppose that decay occurs as an asset ages at a constant rate γ. Each year improved assets are introduced to the market at rate θ and assets decay at rate γ. The age-date tableau of relative efficiencies, normalized on a new vintage v=0 asset is illustrated in Figure 1. In this figure I am assuming no changes related to passage of time *per se*. The actual history of any cohort of assets runs down a diagonal starting at age-0 in year-t from left to right. For instance, new period-2001 assets efficiency function throughout their lives is:

$$(1+\theta),\ (1+\theta)(1-\gamma),\ (1+\theta)(1-\gamma)^2,\ (1+\theta)(1-\gamma)^3,\ \ldots$$

Note that each asset cohort decays only, and decay is independent of the rate of obsolescence. This reflects the assumption that the technology embodied in the 2001 assets remains constant as that asset cohort evolves over its history.

Note also that at any point in time, say year 2003, assets of different ages have different levels of technology embodied in them reflecting the assumption of a constant rate of technological change per period. This can be seen by tracing the sequence down a column. For example the 2004 efficiency sequence is:

$$(1+\theta)^4,\ (1-\gamma)(1+\theta)^3,\ (1-\gamma)^2(1+\theta)^2,\ (1-\gamma)^3(1+\theta),\ (1-\gamma)^4$$

Figure 1 Tableau of asset efficiencies by age and date: φ(s,v)

Assuming constant rates of decay γ and of obsolescence θ

Normalized on vintage 2000 new assets: φ(0, 2000) = 1

Age ↓	Date →				
	2000	2001	2002	2003	2004
0	1	$1+\theta$	$(1+\theta)^2$	$(1+\theta)^3$	$(1+\theta)^4$
1	$(1-\gamma)\,(1+\theta)^{-1}$	$1-\gamma$	$(1-\gamma)\,(1+\theta)$	$(1-\gamma)\,(1+\theta)^2$	$(1-\gamma)\,(1+\theta)^3$
2	$(1-\gamma)^2\,(1+\theta)^{-2}$	$(1-\gamma)^2\,(1+\theta)^{-1}$	$(1-\gamma)^2$	$(1-\gamma)^2\,(1+\theta)$	$(1-\gamma)^2\,(1+\theta)^2$
3	$(1-\gamma)^3\,(1+\theta)^{-3}$	$(1-\gamma)^3\,(1+\theta)^{-2}$	$(1-\gamma)^3\,(1+\theta)^{-1}$	$(1-\gamma)^3$	$(1-\gamma)^3\,(1+\theta)$
4	$(1-\gamma)^4\,(1+\theta)^{-4}$	$(1-\gamma)^4\,(1+\theta)^{-3}$	$(1-\gamma)^4\,(1+\theta)^{-2}$	$(1-\gamma)^4\,(1+\theta)^{-1}$	$(1-\gamma)^4$
5	$(1-\gamma)^5\,(1+\theta)^{-5}$	$(1-\gamma)^5\,(1+\theta)^{-4}$	$(1-\gamma)^5\,(1+\theta)^{-3}$	$(1-\gamma)^5\,(1+\theta)^{-2}$	$(1-\gamma)^5\,(1+\theta)^{-1}$

The efficiency function at any point in time is:

$$(11) \qquad \varphi(s,v) = (1-\gamma)^{s} (1+\theta)^{v}$$

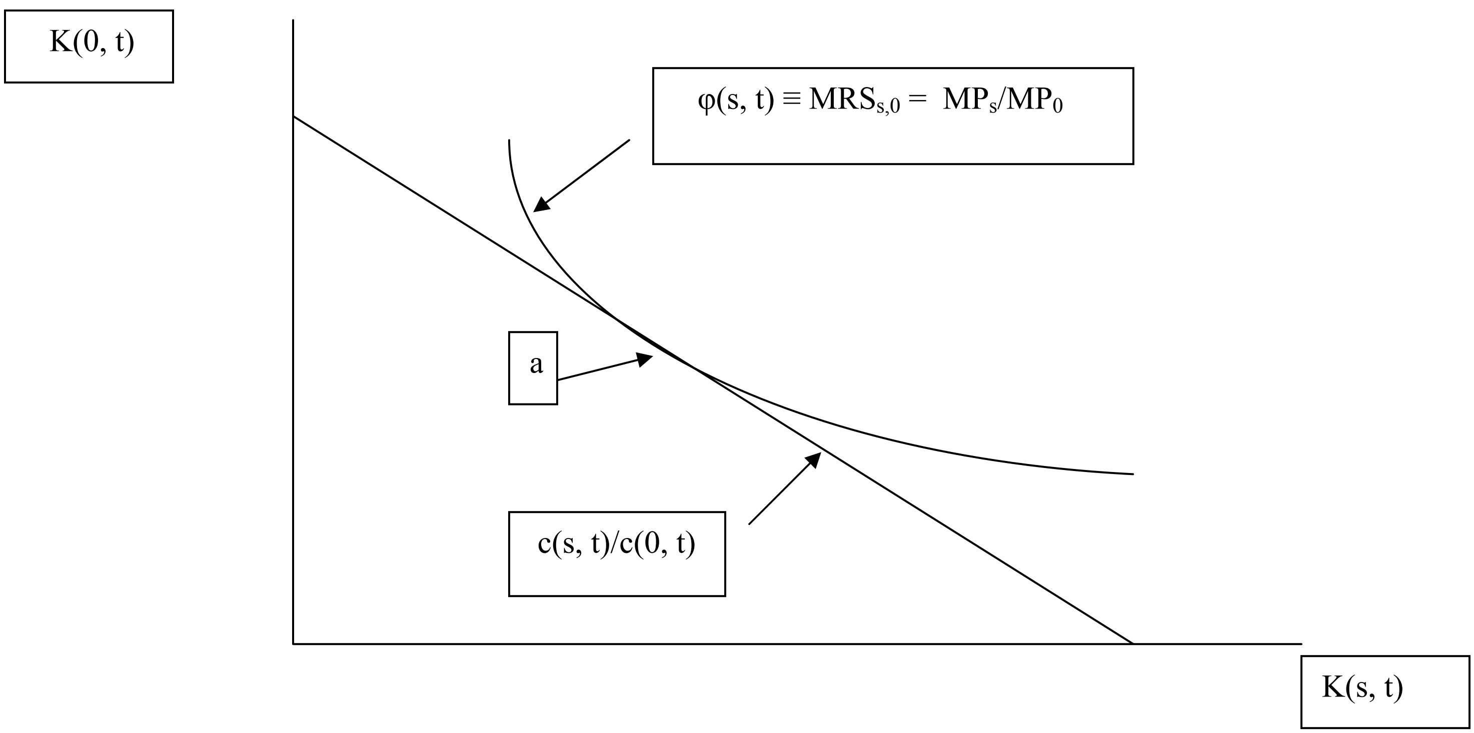
where s = 0, 1, 2, 3, … , v = 0, 1, 2, 3, … , v = t – s.

This function satisfies the conditions outlined for equation (1) for a Jorgenson efficiency function and equation (2) for the mortality function. Note that once the technological innovation raises the relative efficiency of a new vintage asset, the decay rate applies to this higher efficiency relative to the base vintage, v=0, through out asset lives.

Weights for a capital aggregate

We are now prepared to answer the question of what weights should be used in constructing capital stocks from a sequence of investment flows? More specifically, what weights to construct a period-t stock measure from flow data on investments for periods t-s, s = 0, 1, 2, 3, …, S are implied by this Jorgenson model amended for embodied technological change? The efficiency function in equation (11) represents the marginal rates of technical substitution between new and old assets at time t = v + s as illustrated in Figure 2. Jorgenson's duality condition, equation (3) is equivalent to assuming the producer is at point a. However, in principle the assumption in equation (3) is unnecessary, because we are only using the marginal rates of substitution, the φ-function, as weights; and prices are not needed for the story to hold.

Figure 2 The Efficiency Function is the Marginal Rate of Substitution



Using marginal rates of technical substitution as weights the capital aggregate, calculated from past investment flows, I_{t-s}, at time-t is:

$$(12) \qquad K_t = \sum_{s=0}^{S} \varphi(s, t-s) I_{t-s}$$

Since v = t – s, the weights in equation (12) are exactly the relative efficiencies from the equation (11) efficiency function. Note that in measuring capital in period-t we correct used capital for both decay, γ, and obsolescence, θ; not only for decay. Note also that the efficiency function depends both on age and on vintage. This is why this model requires vintage accounts.

The change in the capital stock from period-t-1 to period-t from equation (12) is,

$$(13) \qquad \Delta K_t \equiv K_t - K_{t-1} = \sum_{s=0}^{S} \varphi(s, t-s) I_{t-s} - \sum_{s=1}^{S} \varphi(s-1, t-s) K_{t-s}$$

or

$$(14) \qquad \Delta K_t = \varphi(0, t) I_t - \sum_{s=1}^{S} [\varphi(s-1, t-s) - \varphi(s, t-s)] I_{t-s}$$

Applying equation (11), the efficiency function in the embodied technological change model, to equation (2), the mortality function from the original Jorgenson model for age-s yields,

$$(15) \qquad m(s-1, t-s) = \varphi(s-1, t-s) - \varphi(s, t-s) .$$

Equation (15) is the mortality function for the embodied technological change model. The term on the right hand side of equation (14) in square brackets is m(s-1, t-s), the mortality function applied to vintage v = t-s assets age-s in period-t.

Intuitively equation (14) says that the change in the capital stock from period-t-1 to period-t equals the difference in two terms. The first term is new acquisitions of capital, investment in period-t, I_t, scaled up to allow for embodied technological change from period-0, the base-model, to period-t, φ(s, t). This is the gross additions to the capital stock. We subtract from gross additions the loss in efficiency of old capital by vintage, the mortality rate in the new model, m(s-1, t-s) applied to I_{t-s} for s = 1, 2, 3, …, S.

Several observations from this model are important. All units of capital produced in each period are converted so that in aggregation each is measured in the same units. In equation (12) those units are base-period units of new capital, because we normalized by setting these to 1. In the tableau Figure 1 the base is 2000. One consequence of this base is that throughout its history vintage-2000 capital only decays and looses no value as a result of obsolescence. This is a bit of an illusion however, because the relative weights of assets in each period's capital stock correct both for decay and obsolescence. Consider the ratio of the weight of a new 2004 asset compared to the weight of a 3-year-old asset in year 2004, $[(1+\theta)^4/ (1-\gamma)^3 (1+\theta)]$. The ratio is: $[(1+\theta)/(1-\gamma)]^3$. Each unit of new 2004 capital is worth $(1+\theta)^4$ units of new-2000 capital where as each unit of 3-year-old capital originally was worth $(1+\theta)$ units of new-2000 capital and has since decayed by $(1-\gamma)^3$.

Another way to put this is to use 2004 as weights and redo Figure 1 tableau. See Figure 3. Here the efficiency functions are normalized on 2004 not 2000. Thus, the efficiency function for a new year-2000 asset is now: $(1+\theta)^{-4}$ rather than 1. Year 2000 assets, even though new, are only worth $1/(1+\theta)^4$ units of a new year-2004 asset. The efficiency function of a 3-year-old asset in 2002 is $(1-\gamma)^3 (1+\theta)^{-5}$ rather than $(1-\gamma)^3 (1+\theta)^{-1}$. It has decayed 3 periods independent of vintage, but in new, 2004-vintage assets was only worth $(1+\theta)^{-5}$ in the first place. Capital stocks will be different if we normalize on different years, but that is because they answer different questions. How big, in terms of vintage-v, is the stock of capital? Each vintage-base yields a different answer.

However, the marginal rates of substitution between new and used assets in any one year are the same regardless of the base year. Consider the ratio of the weights between 3-year-old assets in 2002 and new, 2002 assets. Using 2004 base, we have $(1-\gamma)^3 (1+\theta)^{-5}/(1+\theta)^{-2}$ or $[(1-\gamma)/ (1+\theta)]^3$ Using the 2000 base we get exactly the same ratio: $(1-\gamma)^3 (1+\theta)^{-1}/(1+\theta)^2$ or $[(1-\gamma)/(1+\theta)]^3$. The point is that while the aggregates will differ if we use a different period's units as the base, the relative the weights will be the same.

What this model says in effect is that when new technology is embodied in new assets one must recalibrate capital stock measures in order to correct for differences in quality among assets of different vintages. It is not enough to count the number of boxes of crackers sold to get an accurate measure of the number of crackers sold if the number of crackers per box is changing.

Figure 3 Asset efficiencies by age and date: φ(s,v)

Assuming constant rates of decay γ and of obsolescence θ

Normalized on vintage 2004 new assets: φ(0, 2004) = 1

Age ↓	Date →				
	2000	2001	2002	2003	2004
0	$(1+\theta)^{-4}$	$(1+\theta)^{-3}$	$(1+\theta)^{-2}$	$(1+\theta)^{-1}$	1
1	$(1-\gamma)\ (1+\theta)^{-5}$	$(1-\gamma)\ (1+\theta)^{-4}$	$(1-\gamma)\ (1+\theta)^{-3}$	$(1-\gamma)\ (1+\theta)^{-2}$	$(1-\gamma)\ (1+\theta)^{-1}$
2	$(1-\gamma)^2\ (1+\theta)^{-6}$	$(1-\gamma)^2\ (1+\theta)^{-5}$	$(1-\gamma)^2\ (1+\theta)^{-4}$	$(1-\gamma)^2\ (1+\theta)^{-3}$	$(1-\gamma)^2\ (1+\theta)^{-2}$
3	$(1-\gamma)^3\ (1+\theta)^{-7}$	$(1-\gamma)^3\ (1+\theta)^{-6}$	$(1-\gamma)^3\ (1+\theta)^{-5}$	$(1-\gamma)^3\ (1+\theta)^{-4}$	$(1-\gamma)^3\ (1+\theta)^{-3}$
4	$(1-\gamma)^4\ (1+\theta)^{-8}$	$(1-\gamma)^4\ (1+\theta)^{-7}$	$(1-\gamma)^4\ (1+\theta)^{-6}$	$(1-\gamma)^4\ (1+\theta)^{-5}$	$(1-\gamma)^4\ (1+\theta)^{-4}$
5	$(1-\gamma)^5\ (1+\theta)^{-9}$	$(1-\gamma)^5\ (1+\theta)^{-8}$	$(1-\gamma)^5\ (1+\theta)^{-7}$	$(1-\gamma)^5\ (1+\theta)^{-6}$	$(1-\gamma)^5\ (1+\theta)^{-5}$

Producer behavior: Obsolescence vs. deterioration

Until now we have avoided the implications of the behavior of producers whose used capital becomes obsolete for capital measurement. A long tradition in capital theory has been to think that obsolescence, as opposed to deterioration, should not lead to a lower weight on old assets. Joan Robinson and Robert Solow agreed that, in Robinson's terms, "an overcoat is an overcoat," meaning that the old capital retains its worth as new, superior vintages enter the market.[3] In more recent years, Zvi Griliches argued that older vintages of capital retain their in-use productive efficiency when better vintages of capital enter the market. Griliches illustrated his point by observing that even if new material is introduced by new vintages of faculty, the material taught by older tenured faculty is still just as good as he was (provided new research has not shown that what he is teaching is wrong.) Is this view correct?

The Griliches Assertion

The conventional wisdom means that unless the old assets wear out (decay), they retain their full value even though they are now obsolete relative to new ones. One implication of this view is that producers who employ used capital respond differently to obsolescence than they do to deterioration. A decline in price resulting from deterioration lowers an asset's value, but a decline in price resulting from obsolescence does not. The argument is that in the case of deterioration, the old asset has physically decayed and is less productive. However in the case of obsolescence, the old asset is physically unchanged in productivity and thus should not be reduced in value. This means in turn that even when we observe a decline in the market price of used assets as a result of obsolescence, their in-use value or productive efficiency, remains the same.

A related question is: If Griliches is correct and one must distinguish obsolescence from deterioration in valuing capital is the Jorgenson's duality condition wrong? Clearly obsolescence reduces the relative price of used assets to new. Does it also lower the in-

[3] Michael Harper made this point an NBER summer workshop in July, 2003.

use efficiencies represented by the Jorgenson efficiency sequence? After all these are two important ideas, sacred cows perhaps, in capital theory. Can they coexist, or do we have to choose one over the other? I will analyze this question first assuming duality; then I will drop the duality assumption to explore the consequences.

Decisions by a Jorgenson producer

Consider a rational producer initially at a competitive equilibrium, like point-a in Figure 2. This equilibrium satisfies Jorgenson's duality condition equation (3), in which the ratio of the user costs equals the marginal rate of substitution between used and new assets. In equilibrium this result holds for all assets, and for the labor as well as other inputs.

(16) $MRS_{s,0} = c(s)/c(0) \quad MRS_{L,0} = w/c(0) \quad MRS_{x,0} = p_x/c(0)$

L is labor and x is other inputs. MRS is the marginal rate of (technical) substitution. Using marginal products, MP, equation (16) also means that

(17) $MP_0/c(0) = MP_s/c(s) = MP_L/w = MP_s/p_x.$

The marginal product per dollar (the numeraire) spent on each input is equal to any other.

We are going to consider how a rational producer under several different sets of circumstances will react to new (presumably superior) technology embodied in new capital. First, however, we should be more explicit about what we mean by technological change embodied in new capital. Does the technological change lower capital costs? Does technological change increase marginal product? Does technological change lower marginal product and increase capital costs or do capital costs stay the same or even fall when the marginal product of new capital increases? Does the technological change per se alter the marginal products of other inputs or leave them unchanged? Is it possible for technological change to increase the average product of new capital but not the marginal product? In the abstract technological change could involve many of these alternatives and we have to be careful to explicate which alternatives are relevant to our analysis.

Economists have long considered the possibility of skill-biased technological change. Economists have also considered models in which new technology is exogenous to the economy and leaves the production relation in tact: New technology increases output for each initial combination of inputs without altering their marginal rates of substitution and it does so without an increase in social costs. Technological change of this type is **Hicks neutral** technological change that is also **manna from heaven**. Of course, since it is Hicks neutral it cannot be embodied in new capital. If a technological change is to be embodied in new capital, then it must have one of two properties: It must either change the physical nature of new capital relative to earlier vintages of the new capital, or it must lower the cost of using new capital relative to the cost of earlier vintages. Either of these properties rule-out Hicks neutral technological change.

The manna from heaven property is a little different. If the marginal physical product of new capital rises but the costs stays the same, then this is compatible with manna from heaven. What if the physical nature of the product is the same but costs have fallen? Could this be manna from heaven? Yes it could be, but this would mean the manna has fallen somewhere else—the cost of some input into the production of this capital has fallen but the capital itself is unchanged. We consider either type of technological change—the new capital is the same but less costly to the user, or at the same cost, the new capital is more productive.

The manna from heaven property is not necessary however. In endogenous growth models, by definition, technological innovations have social costs. To what extent are these costs passed on to the buyer of new capital in which new technology is embodied? As Hulten has pointed out the social welfare implications of socially costly innovations as opposed to manna differ. But, here we are considering how a user of capital, who holds old assets, will react to new technology embodied in new capital but unavailable to his used assets. (He cannot retrofit this new technology onto his old capital.) If the capital-user incurs the full costs of the new technology then I would argue that, from his perspective it is not a technological innovation. If the cracker box has one third more crackers but costs one third more, then from the buyer's point of view the new box is no better than the old.[4] Only if the innovation results in a better product per unit of costs does the producer, who uses the capital, encounter a technological innovation in any meaningful sense.

Technological change embodied in new capital rules out **Harrod neutral** technological change, because Harrod neutral technological change augments the labor input without altering the capital input. Since both old and new capital are by definition included in the capital input the capital input must change. In fact, it is not adequate to simply multiply the stock of capital by some technological growth rate, because the measure of the capital aggregate must itself change. The K-function in equation (8) must change.

Finally, suppose a new invention increases the average product of new capital. Could this occur without altering the marginal rate of substitution between new and used capital? We are considering here technological change embodied in new capital in the sense that the new capital is superior to its older vintages. We mean superior in the sense that the marginal product per dollar is higher than a new vintage was in the previous period. This means that either the new asset cost less to use or it produces more per unit of cost in comparison to old capital. In comparison to the previous period's capital, the product of the new one asset is higher. If the average product has increased then assuming the same marginal product for the old capital, the marginal rate of substitution between new and used capital must have decreased—the ratio of the marginal product of the new asset per dollar must have risen relative to that of used assets.

Whether this lowers the cost per unit of new capital or raises its marginal product, embodied technological change lowers the cost per unit of marginal product of new capital. The result is that the marginal product per dollar of new capital has increased.

[4] I believe this is the case that Hulten has in mind when he argues that unless there manna from heaven will be treated differently than new technology with exactly offsetting increases in costs.

Assuming no decay and no deterioration, we will assume the ratios of the other inputs, old capital and labor, all remain unchanged,[5] so that,

(18) $MP_0/c(0) > MP_s/c(s) = MP_L/w = MP_x/p_x$.

How does the producer respond to these new circumstances? In a competitive equilibrium in which all inputs are variable, which is the Jorgenson model, the producer will substitute away from age-s capital, labor and x-inputs toward new capital. The substitution continues until the equilibrium condition equation (17) is re-established. The result will be an increase in the marginal product per dollar of used assets (and other inputs) **relative** to the marginal product per dollar of new assets. It is at the margin that obsolescence results in a substitution away from used assets and other inputs toward new assets. The marginal product of age-s assets rises or the user-cost of an age-s asset falls or both occur. This means that either the quantity of used assets demanded falls or the asset price of used assets falls.

It would appear that in the Jorgenson model, in which duality holds, the value of the used stock of assets falls as a result of obsolescence. I would conclude that Griliches' assertion does not hold up in the Jorgenson model. Of course one may ask: Is the Jorgenson model reasonable? Perhaps Griliches is right. If so, then should we scrap the Jorgenson model? On the one hand, the Jorgenson model is coherent even if it is not consistent with Griliches' view. On the other hand, many critics dislike the equilibrium duality condition arguing that producers are locked into age-s assets. However, an implicit assumption of the Jorgenson model is that producers can substitute new assets for old assets. This is certainly true in some, perhaps many, cases.

[5] If all the equalities in equation (18) hold, then we have ruled out skill biased technological change. Since we are interested here is how the producer acts regarding new versus used capital, the possibility that new capital increases the marginal rate of substitution between two other inputs is not pertinent. Skill biased technological change refers to an innovation that alters the relative value of skilled and unskilled workers. This could take place or not and the marginal rate of substitution between new and used capital would still have increased.

If used-asset markets exist, then producers can sell off old assets in the used-asset market. The used market can be more subtle than often acknowledged by critics. Tax breaks, rebates, pollution credits and other inducements may be available which make “resale” of used assets more likely. In the case of cars for example old cars can be donated to charities for tax deductions. In the case of computers, firms may obtain tax benefits by donating used computers to schools. Many used assets are also exported from the US to other countries for various secondary uses. Furthermore, secondary uses for old capital may exist within large firms themselves. In fact all that is necessary is the presence of a down stream use for the old asset. Such a down stream use implies that the old asset is less productive as a direct result of obsolescence. Since many opportunities are available to unload old assets on used markets, the Jorgenson duality condition, while not universal, may be a reasonable approximation of reality.

Decisions assuming no used-asset markets

What if used markets do not exist and producers cannot receive market value for their old assets? In Griliches’ famous example, senior Harvard professors are tenured so that even if superior young professors, new vintages, are available, President Summers of Harvard cannot unload his senior faculty in order to hire the new young Ph.D.s. If a person owns a two year old computer, he may not be able to sell it at a price offsetting its in-use value. This would be true in an Ackerlof lemons model for example.[6] What do producers do when they are stuck with the used asset?

Suppose we drop the assumption that used capital is a variable input and treat the old asset as a sunk cost. If the producer does not use it, he may as well throw it away.[7] Does this mean that the Griliches assertion is correct—the in-use value of old capital is

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[6] Ackerlof (1970) argued that asymmetric information in which buyers are ignorant of the quality of a used asset results in a market dominated by lemons. Owners of cherries cannot get value in such a market. Of course, the logical extension of the Akerlof model is a world without used asset markets. As Akerlof himself explained owners of used cherries who wish to sell them have an economic incentive to either inform potential buyers or provide a guarantee on quality. Such behavior can be observed in used-asset markets which are often quite vigorous.

[7] I use throw away at no cost here to avoid complications of scrapping old assets for value. I will consider a model in which costs (or possibly benefits) accrue from scrapping old assets.

unchanged? How will the producer behave if the used asset is a sunk cost? In this short run, the rational producer will only operate at the margin available to him. While he cannot recapture his sunk costs, he does have a number of margins on which to operate. His variable costs include labor and other variable inputs that are used along with capital in production. These would include fuel, maintenance and repair, electricity, ink, and software to name a few. All of these are possible substitutes for new capital when the marginal product per dollar of new capital has risen.

This explains why producers may reduce their secretarial pool as they bring new computers with embodied technological improvements into use. It explains why less skilled workers are laid off when new capital, with a lower marginal product per dollar becomes available. It also explains why maintenance and repair expenditures are increasingly replaced by total replacement of damaged used machines for new ones.

This still does not quite explain how the producer decides when to abandon the old machine. He will abandon the old machine when the marginal, variable costs of using the old machine exceed its in-use marginal benefit. The marginal benefit of using the old machine is its marginal product. His decision rule then is to continue to use the old machine as long as the following holds,

$$(19) \qquad VMP_Q - MC_z \geq 0 .$$

$VMP_{s,}$ the value of marginal product, is product price times the marginal physical product of age-s capital and MC_Z, the marginal cost of variable inputs used with age-s capital in producing output, is the sum of the variable input prices times quantities used.

Is this decision rule influenced by technological change embodied in new capital? While it may not appear to be so, it is. Variable costs of using the old machine include the opportunity costs—the forgone earnings from using the variable inputs at their next best alternative; namely, in our example, matching up continued use of the old machine to acquiring the new machine. Still, the new machine must be purchased[8] and the residual cost of the old machine cannot be realized. When does this producer decide to retire his old machine and replace it with a new one?

The producer will replace the old machine with the new machine when the value of old capital's marginal product minus the variable costs of inputs used in tandem with the old machine is less than the marginal product of the new machine minus both the costs of the old inputs working with new capital plus the user-cost of the new capital.

$$(20) \qquad VMP_0 - [c(0) + MC_z^0] > VMP_s - MC_z^s \, .$$

Since product price, p, is unlikely to change, equation (20) can be written as:

$$(21) \qquad p(MP_0 - MP_s) - (MC_z^0 - MC_z^s) > c(0)$$

or, if input prices other than new capital remain fixed, then,

$$(22) \qquad p\Delta MP_{0,s} - \sum_z p_z \Delta z > c(0)$$

where Δ represents the difference between new and used marginal product in the case of MP and quantity of ancillary inputs in the case of z.[9] Fewer z-inputs may be required for use with the better new machine, because embodied technological change may include lower ancillary costs, such as improved energy efficiency. If, however, the same ancillary inputs are required to use the new capital, then the condition is:

$$(23) \qquad p\Delta MP_{0,s} > c(0)$$

[8] By assuming that used capital is a sunk cost we are not only assuming no resale markets for used assets but also no used-asset rental markets.
[9] Once this marginal condition is met, it should continue to be met during future periods of use provided efficiency sequences decline with age (or use).

The increase in the value of marginal product from switching to new capital must exceed the user-cost of the new capital. Clearly technological change embodied in new capital will influence the decision to stop using old capital even when old capital costs are sunk costs![10]

Obsolescence vs. deterioration revisited

So far we have analyzed the decision process in the short run and the long run of a producer who has to decide how to respond when new capital embodies technological advances over used capital. In each instance, under the Jorgenson model and under irreversibility, we have established the conditions in which a rational competitive producer will substitute new for used capital. We could go on to consider other market models, monopoly and so on, but this would not seem to change the substance of the key argument.

In particular, this analysis of producer behavior has an important implication for the assertion that the quantity of old capital in stock measures remains unchanged when new vintages of capital embody technological change. Evidently the decisions made by the rational producer are always made at the margins—the only difference between the short and long run is which margins are operable. In each case it is comparisons of marginal product to price ratios of different inputs that bear on the decisions. In the short run if condition in equation (18) holds then the producer switches to new capital. In the long run if the condition in equation (23) holds then the producer switches to new capital. In both cases, it is the ratios of marginal products to user-costs that matter.

In the mental experiments above the relative ratios changed as a result of technological improvements embodied in new capital making old capital obsolete. However, what if the initial change had resulted from decay of used capital rather than from obsolescence? It doesn't matter, because the rational producer based his decisions only on changes in

[10] Dividing both sides of equation (23) by c(0) results in $p\Delta MP_{o,s}/c(0)>1$. The left hand side of this expression is Tobin's q. If the incremental benefit of new capital exceeds the replacement cost then the producer will invest in new capital.

ratios regardless of how they came about. A decrease in the marginal product of used capital as a result of decay, reflected on the price side by deterioration, acts the same as an increase in the marginal product per dollar of new capital as a result of obsolescence. I conclude that the obsolescence-deterioration distinction is a red herring.

Implications for Utilization

The above analysis also has implications for adjusting capital stock measures to obtain capital flow measures by allowing for utilization. Berndt, Mori, Sawa and Wood (1990) develop and apply relative utilization weights of different vintages of capital in order to assess the substitutability consequences of the energy price shocks during the 1970s. They show that the US manufacturing sector lowered utilization rates of energy-intense old capital in response to price increases of gasoline. The question I now address is in measuring capital stocks what allowance should one make to allow for variations in utilization rates of different vintages of capital. To make sense of this question I distinguish between resale, retirement, and scrap of used assets. As above, I also consider the short run vs. the long run in which a rational, competitive producer decides how to deal with his used capital.

Equation (21) shows the criteria on which the producer replaces old with new capital when used-asset markets are unavailable so that used-asset costs are sunk. The second term on the right hand side of equation (21) is the difference between the costs of ancillary inputs that must accompany new capital vs. used capital. If new technology results in less energy-intense inputs and energy costs have risen then, as Berndt, Mori, Sawa, and Wood show, the producer is induced to substitute new energy efficient assets for old.

However, in our model producers replace the old asset in its current use with a new asset. The used asset is either resold or, if possible shifted to a lower-valued use. This suggests that the old assets are still in-use but in a different activity and that this distinction is reflected in its lower user-cost. In this case, the utilization rate of old capital does not fall.

Its use shifts and the user cost, and consequently the asset price, will decline to reflect this fact. It would be redundant and misleading to correct in addition for variations in utilization.

Of course, at some point down stream the net benefits of an old asset in any use will be less than scrap value and the asset will be scrapped. At this point the asset is retired from service and will no longer appear in the stock. Its weight will have dropped to zero as has its user-cost. This is the retirement case and of course one must allow for retirements.[11]

None of the models considered so far implies lower utilization of old capital per se. Each producer is generating output with one of two technologies. One uses new capital and the other old capital. Each chooses one or the other.

One could establish circumstances in which a producer will under-utilize old assets. In theory, the circumstances would seem to consist of an unexpected change in asset-use conditions when limited options are available to producers and when producers become less certain about future conditions that influence capital inputs. This type of circumstance is exactly the one envisioned and modeled by Berndt, et al. A shock hits energy-intense capital users and leaves them in a less certain environment. It is hard to imagine that capital users in the US had anticipated the decision by OPEC to instantaneously quadruple energy prices. Furthermore it is hard to imagine that firms were able to predict, with the degree of confidence they had before the shock, the path of energy prices after OPEC formed and jacked up price for the first time. A sensible reaction of capital users under these conditions is to wait until the future becomes less obscure. In this case the service flow from old capital will fall relative to that firms had expected to employ. This means that a distinction be made between the available service flows reflected in a priori user-costs will have to be corrected for new utilization rates.

[11] See Hulten and Wykoff (1981) for a coherent treatment of retirements albeit one calling for limited information on retirements. The limitation reflects the data available for measurement purposes at the time..

However, these conditions are unlikely to persist. They are, by their very nature, transitory—expectations were wrong and confusion rose. Even if expectations are not formed rationally, eventually producers will adjust their expectations, correct their forecasts (and standard errors about those forecasts), and capital asset prices and user costs will again reflect normal conditions. It seems to me that at this point adjustment for utilization should not be employed. Capital stock measures intended to indicate normal conditions, even normal swings in economic activity, business cycles, should not be adjusted for a distinction between service flow and user-cost. At the margin, these are set equal by rational producers.

If over the course of the business cycle, demand unexpectedly falls short, then capital utilization will decline. Even here surely firms in a modern capitalist economy do not find business cycles *per se* unexpected. Ordinarily then, even over the business cycle, a reasonable model underlying measurement of capital would assume that capital operates pretty much as expected. It is only unpredictable surprises followed by a period of increased uncertainty, that drive capital-use away from expectations and therefore away from costs—Keynesian surprises, followed by increased uncertainty about future events, result in utilization rates below that implied by efficiency functions that equal user cost ratios.

This analysis implies that utilization indicators are useful devices for analyzing business cycles resulting from sudden surprises and increased uncertainty. Furthermore, retirements must be built into depreciation estimates. However, utilization rates should not be used in measuring the capital input in a neoclassical framework and it is this framework that, Joan Robinson and P Sraffa notwithstanding,[12] underlie capital measures in national accounts.

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[12] See Harcourt and Laing (1972) which contains articles both by Robinson and Sraffa.

Conclusion

The first half of the paper shows that the measurement of capital over time must correct for both deterioration and obsolescence in order to reflect a coherent aggregate—that is, aggregating in like-units. The last half of the paper establishes conditions in which owners of used assets will switch to new assets. The argument is that the decision making process of a rational producer will depend on the same marginal conditions regardless of whether they are changed by obsolescence or deterioration. If the producer is restricted on some margin in the short run then that restriction will take place whether or not his old asset has decayed or become obsolete! Only under unusual circumstances should the in-use services from capital deviate from the efficiency sequence as a result of variations in utilization. The behavioral model also suggests that a sound assumption under ordinary circumstance is that relative user costs equal in-use productive efficiencies, so that the duality condition is a sound lynchpin of capital measurement.

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