Chain Price and Volume Aggregates for the System of National Accounts

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The introduction of a new commodity ought, evidently, to change, in some degree, any price index which pretends to be a sensitive expression of the data from which it is computed (*unless*, of course, *the new commodity happens to have a price relative exactly equal to the index number*). [Italics are Baldwin's]
Irving Fisher, *The Making of Index Numbers*[1]

1. Summary and Introduction

This paper relates to the recommendations contained in the *Systems of National Accounts 1993* on volume measures of gross domestic product (GDP). These recommendations are contained in Chapter XVI, which was written by the distinguished English economist Peter Hill. Perhaps the best way to summarize this paper would be to list Hill's own recommendations followed by the amendments proposed herein:

(a) Original: The preferred measure of year to year movements of GDP volume is a Fisher volume index; changes over longer periods being obtained by chaining; i.e., by cumulating the year-to-year movements.
Amended: The preferred measure of year-to-year movements of GDP volume is an Edgeworth-Marshall[2] volume aggregate; changes over longer periods being obtained by chaining; i.e. by cumulating the year-to-year movements.

(b) Original: The preferred measure of year to year inflation for GDP is, therefore, a Fisher price index; price changes over longer periods being obtained by chaining the year-to-year price movements: the measurement of inflation is accorded equal priority with the volume movements.
Amended: The preferred measure of year to year inflation for GDP is an Edgeworth-Marshall price index. However, as the Edgeworth-Marshall formula does not satisfy the strong factor reversal test, direct and chain implicit price indexes should also be calculated and published. The direct implicit price index would be based on the ratio of the expenditure series at current prices to the expenditure series at constant prices (i.e. the weighted

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[1] quoted by Krtscha, p.136

[2] I have referred to index formulas by both their co-authors if they were co-discovered, with the authors listed in alphabetical order. Thus, Edgeworth-Marshall instead of Edgeworth or Marshall-Edgeworth, Montgomery-Vartia instead of VartiaI and Sato-Vartia instead of VartiaII, Bowley-Sidgwick instead of Sidgwick. Bowley was the propogandist rather than the discoverer of the Bowley-Sidgwick formula, but he should share the credit for it since Sidgwick did little more than note the possibility of an arithmetic cross between Laspeyres and Paasche formulas, whereas Bowley somewhat advocated it. However, the Fisher formula remains the Fisher formula, even though others, including Bowley, wrote about it before he did.

average of prices over two years). For highly cyclical commodities, a modification of the Edgeworth-Marshall formula will be required for the inflation indicator, and the basket reference period can span three to five years as required. (This may also be necessary for the cyclical components of the GDP volume measure.) In the case of seasonal commodities, the price index would depart from this formula to incorporate a seasonal weighting pattern, preferably using the Rothwell or Balk formula.

(c) Original: Chain indices that use Laspeyres volume indices to measure year-to-year movements in the volume of GDP and Paasche price indices to measure year-to-year inflation provide acceptable alternatives to Fisher indices.

Amended: Omitted. This recommendation is unneccessary as the Edgeworth-Marshall chain-volume aggregates are chain fixed-price volume aggregates, like the Laspeyres volume aggregates, but without their notorious penchant for chain drift.

(d) Original: The chain indices for total final expenditures, imports and GDP cannot be additively consistent whichever formula is used, but this need not prevent time series of values being compiled by extrapolating base year values by the appropriate chain indices.

Amended: The chain volume aggregates for total final expenditures, imports and GDP will continue to be additive because they will be linked backward and not forward. For years prior to the most recent two or three years, the chain volume aggregates cannot be additively consistent whichever formula is used. For this reason there should be overlapping volume series covering spans of at least four to six years based on one set of constant prices for the central two years of the span. Similarly, there should be overlapping fixed-basket price indexes covering spans of at least four to six years using the central two years of the span as the index basket. For statistical agencies that would find the data or computational requirements of annual chaining too onerous, chaining every four to five years of volume series whose constant prices are based on the central year or years of the span would provide a cheaper, acceptable alternative. Similarly, price indexes could be calculated with chaining every four to six years with a basket reference period based on the central year or years of the estimation span.

(e) Original: Chain indices should only be used to measure year-to-year movements and not quarter to quarter movements.

Amended: Chain volume aggregates and price indexes should be calculated both annually and quarterly. The fixed-price structure of the volume aggregates (i.e.using average prices over two years) will apply to both quarterly and annual series, ensuring that meaningful measures of quarterly volume change can be derived for all consecutive quarters for the direct series, and for all but the Q4-to-Q1 movements of the chain series.

The most controversial of these proposed amendments may well be (a) and (b). Some economists would reject the Edgeworth-Marshall formula because it is not exact for any utility function. If the choice of formulae were limited to those defined as superlative, then (a) and (b) can be rewritten to replace the Edgeworth-Marshall formula with the linear Walsh formula, which is exact for the Generalized Linear utility fuction. Because of its matrix consistency properites, it would still be a better choice than the Fisher formula. This has already been suggested by Diewert(1995) in his critique of Hill's paper.

However, the Edgeworth-Marshall formula would seem to be the better choice, as unlike the Walsh formula (or the Fisher formula) it respects the property of transactions equality (i.e. the importance of a transaction in the formula does not depend on the period in which it occurs.) Also, unlike the Walsh formula, it does not simply discard commodities from a volume aggregate if they go to a zero price from a positive price or vice-versa, nor does it simply discard commodities from a price index if they go from a positive quantity to a zero quantity or vice-versa.

2. The SNA68 Volume Measures

In *A System of National Accounts* (1968 SNA) the volume measures that were prescribed were Laspeyres volume aggregates. These would be direct measures over the recent period, although they would be linked backward over their historical period. Thus additive consistency would be preserved for the most recent history of the volume series but not, for a given base period, over the entire series.

Some features of the direct Laspeyres volume aggregates are worthy of note.

First, they are Laspeyres volume aggregates, rather than Laspeyres volume indexes, because they are expenditure totals and not index numbers, with the formula:

$$\sum p_0 q_t$$

(Summation is over commodities, but to reduce the burden of notation, the commodity superscript is omitted. In the rest of the paper, summation is assumed to be over commodities unless otherwise indicated.)

The Laspeyres volume aggregates can be defined as measuring period t expenditures at base year 0 prices. When these aggregates are indexed to base year expenditures, one gets:

$$Q^L_{t/0} = \sum p_0 q_t / \sum p_0 q_0$$

a Laspeyres volume index. However, in actual statistical practice, such aggregates are not commonly indexed, it being much more common to publish expenditure series at constant prices than Laspeyres volume indexes.[3]

Second, component by component, the base year price is defined so that when multiplied by the quarterly quantities for the base year and aggregated, the total should equal base year expenditure, i.e.

$$\sum_j p_{0q} q_{0q} = \sum_j \bar{p}_0 q_{0q}$$

which requires that

$$\bar{p}_0 = \sum_q p_{0q} q_{0q} / \sum_q q_{0q} = \sum_q p_{0q} \left(q_{0q} / \sum_q q_{0q} \right) \qquad (2.0)$$

that is, the base price is calculated as the unit value of all base year transactions, which is a weighted arithmetic mean of quarterly prices, weighting by quarterly quantities. This is really the uniquely optimal estimate of the average price for any homogeneous commodity. This would seem to be so obvious that one should be able to just state it and move on, but since some economists would define base prices over two years in a different way maybe this point should be underlined. Suppose that exactly the same transactions occurred two years in a row at the same set of prices, one would want to budget the same amount of money for these expenditures in both years. Suppose further that the budget for year 1 is established as the number of units purchased multiplied by the average price in year 0. If one chose an average price less than the unit value one would not have enough funds to make all the transactions required. If one chose a price greater than the unit value then one would allocate more funds to these transactions than was necessary and end up with a surplus, which would be greater the larger the discrepancy between the unit value and the overestimated annual price. The same would be true if there were an increase in the volume of transactions but the seasonal profile of volumes and prices remained constant.

It can be seen that if the production of a commodity were completely inelastic with respect to price, all of the quantities in (2.0) would be equal, and the unit value would reduce to an arithmetic mean of prices, which always exceeds the geometric mean of prices.

[3] Until recently, the only important volume index calculated by the CSNA was the index of industrial production. Even for this series, the CSNA now publishes a chain volume aggregate, rather than a volume index.

A mean of quarterly base prices, weighted by quantities, the base year unit value can also be interpreted as a harmonic mean of prices, weighted by expenditures, that is:

$$\bar{p}_0 = 1/\sum_q w_{0q}(1/p_{0q}) \text{ where } w_{0j} = p_{0q}q_{0q}/\sum_q p_{0q}q_{0q} = v_{0q}/\sum_q v_{0q} \qquad (2.1)$$

and for empirical purposes, this is the more useful interpretation. It can be seen that if production of the commodity were unit-elastic with respect to price, so that whatever the change in price, the same revenues were generated, the base year unit value would reduce to a simple harmonic mean of quarterly base prices, which is always inferior to the geometric mean of base prices. Thus the unit value is not consistently higher or lower than the geometric mean, being higher for commoties with higher price elasticities and lower for commodities with lower price elasticities.

In actual national accounting practice, one would rarely have actual price and quantity data to work with. Instead one would be dealing with value series and independently derived price indexes that serve as deflators. The requirement replacing (2.1) would be that:

$$\sum_q v_{0q} = \sum_q v_{0q} / P_{0q/0} \qquad (2.2)$$

where $P_{0q/0}$ is the price index for a given quarter of year 0 with base year 0. Generally, the identity in (2) does not hold and it must be forced, that is, the quarterly values for the base year will be prorated so that the value for year q will be:

$$f \times v_{0q} / P_{0q/0} \qquad (2.3)$$

where $f = \sum_q v_{0q} / (\sum_q v_{0q} / P_{0q/0})$

(Strictly speaking, the adjustment factor *f* should be applied to all quarterly values, but in practice it is usually applied only to the base year quarterly values, creating a slight discontinuity between the base year estimates and the estimates that follow.)

Third, the ratio of the expenditures at current prices to the expenditures at constant prices provides an implicit price index that has the Paasche formula:

$$P^P_{t/0} = \sum p_t q_t / \sum p_0 q_0$$

which can serve as a price indicator for GDP. In production, Paasche price indexes offered official statisticians the choice of calculating volume aggregates directly or by deflating using the Paasche deflator. In terms of calculating seasonally adjusted series, the link was important because volume aggregates were usually calculated indirectly as seasonally adjusted series at current prices deflated by raw or seasonally adjusted Paasche price indexes.

However, the importance of this association of Laspeyres volume aggregates with Paasche price indexes should not be exaggerated. A Paasche price index is a poor indicator of price change except for binary or two-period comparisons since quarterly price changes are always distorted by changes in basket. In Canada, dissatisfaction with the Paasche deflators led to the development of Laspeyres price measures and later chain Laspeyres series, years before the current chain volume measures were introduced.

Fourth, there is additivity of the components of GDP in the volume aggregates, as one would expect since they are simply the expenditures of each quarter at a common set of prices. Expenditures on consumer goods and on consumer services sums to total consumer expenditure, construction expenditures and investment in machinery and equipment sums to gross fixed capital formation, and so forth.

There is also additivity of the monthly or quarterly GDP estimates, which sum to equal the annual estimates, and the same applies for all major components of GDP.

Finally, there is at least the possibility with this kind of structure of additivity of provincial or regional GDP estimates to the national total. In Canada, this was imposed, so that the provincial GDP estimates by industry at 1997 prices summed to the all-Canada totals.

This multi-dimensional additivity over commodities, industries, months or quarters, and regions is better characterized as matrix consistency. A desirable feature of GDP volume aggregates is that they be additive along any dimension of a matrix of values at constant prices.

Fifth, the change between any two quarters or any two years of the volume aggregates satisfies the proportionality test: if all of the quantities in a given period are k times the corresponding quantities in the comparison period then the volume aggregate shows a k-fold increase between the two periods. As a special case of this, the volume aggregates satisfy the identity test: that is, if all the quantities in a given period are identical with the corresponding quantities in the comparison period, then the level of the volume aggregate will be the same in the given period as in the comparison period. (The identity test is just the proportionality test for k=1). This is a consequence of all expenditures for all periods converted to the same set of fixed prices.

Sixth, the Laspeyres volume aggregates are consistent in aggregation, that is, they can be equally well calculated from basic components or from higher level subaggregates.

Seventh, the Laspeyres volume aggregates have the new commodity property alluded to by Irving Fisher in the opening quotation of this paper. If a new commodity is added to the volume aggregate with the identical volume movements as the aggregate, the movement of the aggregate will not change. Although this seems like a banal property, it is quite useful in actual statistical work since frequently one may have weighting information of some kind for a new commodity for a reference year but not detailed price or quantity data. It was possible to create a volume series for the new commodity, explicitly or implicitly by imputing the group movement in a straightforward way. Also, one would immediately know, if detailed data were available for a new commodity, if it would push up the aggregate movement or drag it down, no small matter for agencies concerned with making their GDP by industry and GDP by expenditure category estimates mesh, where the implementation of a methodology change in a given revision cycle could depend on whether it tended to increase or decrease an aggregate growth rate, and decisions are often made under severe time pressures.

Eighth, *SNA68* recommended rebasing of the volume aggregates every five or ten years. Although the manual is quite vague about the mechanics involved, it is clear that it favoured retaining the same movements of historical series when the relative price structure was updated for the recent period. It is agnostic on the question of how far back a new set of relative prices should be carried in calculating volume aggregates.[4] The two obvious choices are to:

1. Calculate volume aggregates at new base year prices starting from the base year itself, which has been the Canadian practice.
2. Calculate volume aggregates at new base year prices starting from two years prior to the new base year for rebasing every five years, which has been the British practice.

The first procedure reduces the magnitude of the revisions of the volume aggregate series, but generally at a heavy price in their representativeness. Even with updatings of the base year every five years, the finalized estimates for the last year of a span will be four years removed from their reference year, versus two years using the second procedure. Overall, the second procedure would seem to be superior.

The historical volume aggregates would be linked to the new volume aggregates using a rebasing factor for each series, as follows:

$$(\sum p_5 q_5 / \sum p_0 q_5) \times \sum p_0 q_t ; t = 0,1,\ldots,4.$$

where expenditures at year 0 prices are linked to the new series at year 5 prices. Typically, there would be a loss of additivity when this rebasing was carried out for the historical period (e.g. the sum of expenditures on consumer services and consumer goods would no longer equal total consumer expenditure). In Canada, this was handled with an elaborate set of adjusting entries that would reestablish additivity between the sum of components and their aggregate, although it would probably have been simpler just to publish a note of warning that due to chain linking, additivity did not hold.[5]

3. The Major Weaknesses of the SNA68 Measures

The principal weakness of the *SNA68* methodology was obvious to its framers even at the time when it was promulgated. In a rapidly changing world a set of constant prices can get out of date after a five year interval, and will be unrepresentative of the economy whose output it is supposed to measure. Thus, *SNA68*-type estimates frequently lacked the quality of representativeness. Also, given a negative correlation between prices and quantities, there will tend to be a positive bias in direct Laspeyres volume measures, which will be more serious the longer a single set of constant prices is retained.For this reason, *SNA68* itself considered the possibility of chaining volume series every year instead of every five or ten years, but finally rejected the idea for general application because of its onerous data requirements.

A second major weakness of *SNA68* is that the Laspeyres formula does not pass the time reversal test, i.e.

$$Q^L_{t/0} = \frac{\sum p_0 q_t}{\sum p_0 q_0} \neq 1 / Q^L_{0/t} = 1 / \frac{\sum p_t q_0}{\sum p_t q_t} = \frac{\sum p_t q_t}{\sum p_t q_0} = Q^P_{t/0}$$

This means that the Laspeyres quantity index for year t with base year 0 is not equal to the reciprocal of the Laspeyres quantity index for year t with base year 1 as required by the time reversal test. In fact, the reciprocal is the Paasche volume index, which shows the dual nature of the two formulas, one being the complement of the other.

As a consequence suppose that prices and quantities were the same in year 2 as in year 0. A chain Laspeyres volume index, with a link at year 1 would show:

$$\frac{\sum p_0 q_1}{\sum p_0 q_0} \times \frac{\sum p_1 q_2}{\sum p_1 q_1} = \frac{\sum p_0 q_1}{\sum p_0 q_0} \times \frac{\sum p_1 q_0}{\sum p_1 q_1} \neq 1$$

since, in general, year 1 prices will be different from year 0 prices.

Generally one would expect prices and quantities to be negatively correlated, so that

$$\frac{\sum p_0 q_1}{\sum p_0 q_0} \times \frac{\sum p_1 q_0}{\sum p_1 q_1} \phi\ 0$$

[4] “The series which result from this procedure for the period *since the new base was established* will not be of the same form as the earlier series.” (*SNA68*, 4.46). (My use of italics.) If the authors had in mind that the first procedure should be adopted, they would have written “since the new base period”.

The import of this is that a chain Laspeyres volume index may be subject to chain index drift, tending to drift upward due to cyclical correlations between prices and quantities even if the overall level of output between two years at the same point in the business cycle is identical. More frequent chaining will not help matters, but will only aggravate them, by strengthening the negative correlation between the index weights and quantity relatives.

The time reversal test has been dismissed as a useful criterion for price and quantity index numbers because in our world time never does run in reverse, but for statisticians who are more interested in working with appropriate formulae than in clever word play, the failure of the Laspeyres formula to pass the time reversal test constitutes a real problem.

A third weakness of SNA68 is that the volume estimates would not pass the transactions equality test, which states that “the relative importance of each transaction is dependent only on its magnitude”.[6] This is quite obvious, since the relative importance of all commodities in the Laspeyres volume estimates would be based only on the expenditures of base year 0. Base year transactions would consequently have a considerably greater influence on the estimates than those of other years.

This is similar to the property of representativeness already discussed but is not identical with it. For example, if one calculated a 20-year output series with base prices equal to the average annual prices over the entire 20-year period, such estimates would pass the transactions equality test (transactions from every month of every year determining the base prices). However, by taking them from all years, the base prices would be poorly representative of any particular year, particularly the initial and final years.

For a long time, this particular weakness did not particularly bother most national accountants. The emphasis was on finding a base year that was a normal year with a representative price structure. If one were measuring output between 1913 and 1918 for example, one would not feel guilty about using 1913 as the base year, because 1913 was a normal peacetime year and 1918 was an abnormal war year.[7] However, experience has shown that it is not that easy to find a normal year to serve as base. In the CSNA, at any rate, some of the recent base years have proven less than optimal, including 1981 and 1992, both recession years. The year 1981 was especially poor as it was also a year when the relative prices of important commodities were much higher than usual, as were nominal interest rates.

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[5] This was the practice for the quarterly estimates of GDP by expenditure category, but there were never any adjusting entries published for the monthly estimates of GDP by industry.

[6] See Dikhanov, p.3.

In fact, the use of a single year's base prices, whatever year was chosen, can never be appropriate for some industries, notably agriculture, where relative prices are particularly volatile. (It was to avoid the difficulty in finding a single representative base year that 1935-39 base prices were adopted by the CSNA for its initial set of output estimates at constant prices, but subsequently base prices were always based on a single year.)

4. The Natural Remedies of These Weaknesses

In summary the *SNA68* measures had three major weaknesses, lack of representativeness, an upward bias due to the use of the Laspeyres formula, and failure to pass the transaction equality test. All of these can be simply corrected by calculating annually-linked volume series, where each year's expenditures are evaluated at the average of given year and previous year prices. Technically speaking this would involve calculating a chain Edgeworth-Marshall volume series rather than a direct Laspeyres volume series. To make it clear how these measures would differ from the *SNA68* measures, the points from the section on these measures will be rewritten to illustrate the proposed solution.

First, the Laspeyres volume aggregates would be replaced with expenditures at constant prices for years 0 and 1 combined:

$$\sum \overline{p}_{01} q_t$$

The annual index for year 1 for the above series:

$$Q^{EM}_{1/0} = \sum \overline{p}_{01} q_1 / \sum \overline{p}_{01} q_0$$

is an Edgeworth-Marshall volume index.

It can be readily seen that such an index, unlike the Laspeyres volume index, passes the time reversal test:

$$Q^{EM}_{1/0} = \frac{\sum \overline{p}_{01} q_1}{\sum \overline{p}_{01} q_0} = 1 / Q^{EM}_{0/1} = 1 / \frac{\sum \overline{p}_{01} q_0}{\sum \overline{p}_{01} q_1}$$

therefore, chain Edgeworth-Marshall indexes will be much less subject to chain drift than chain Laspeyres indexes.

Second, the base period prices are defined over two years, not one year, providing a more stable reference than a single year's prices would. Commodity by commodity, the base period price is defined so that

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[7] See Bowley, p.92, where this argument is made in the context of a price index, favouring the Laspeyres formula over the Fisher.

$$\sum_q p_{0q} q_{0q} + \sum_q p_{1q} q_{1q} = \sum_{y-0}^{1} \sum_q \bar{p}_{01} q_{yq}$$

which requires that

$$\bar{p}_{01} = (\sum_{y=0}^{1} \sum_q p_{yq} q_{yq}) / (\sum_{y=0}^{1} \sum_q q_{yq}) = \sum_{y=0}^{1} \sum_q p_{yq} (q_{yq} / (\sum_{y=0}^{1} \sum_q q_{yq})) \qquad (4.1)$$

that is, the base price is calculated as the unit value for the two years 0 and 1, the mean of the quarterly prices weighting by quantities. This is, as discussed in section 2, the one and only uniquely appropriate way to calculate the average price of a homogeneous product. This base price can also be interpreted as a harmonic mean of its quarterly prices, with weighting based on expenditures:

$$\bar{p}_{01} = 1 / \sum_{y=0}^{1} \sum_q w_{yq} (1 / p_{yq}) \text{ where } w_{yj} = p_{yq} q_{yj} / \sum_{y=0}^{1} \sum_q p_{yq} q_{yq}$$

As in the *SNA68* case, if one were working with value series and independently derived price indexes rather than prices and quantities, the requirement would be somewhat different. For base prices defined over years 0 and 1, one would require that:

$$\sum_{y=0}^{1} \sum_q v_{yq} = \sum_{y=0}^{1} \sum_q v_{yq} / P_{yq/01} \qquad (4.2)$$

where $P_{yq/01}$ is the price index for a given quarter with a two-year base period covering years 0 and 1. Generally, the identity in (4.2) does not hold and it must be forced, that is, the quarterly values for the base period will be prorated so that the value for quarter q of year y at constant prices for years 0 and 1 will be:

$$f \times v_{yq} / P_{0q/0} \qquad (4.3)$$

where $f = (\sum_{y=0}^{1} \sum_q v_{yq}) / (\sum_{y=0}^{1} \sum_q v_{yq} / P_{yq/01})$

As will be seen in the seventh point, the official chain volume estimates will be based on two year links, so for them there is no issue of whether the adjustment factor should be applied to years 0 and following or only to years 0 and 1. Only years 0 and 1 would be adjusted. However, for analytical purposes, the expenditure estimates at prices of years 0-1 will be calculated from years -1 to 2 or even -4 to 5. Over this span, the adjustment factor should be applied to all years.

Close inspection will show that this calculation of base prices for a two-year base period is otherwise totally analogous with the calculation of base prices for a one-year base period in *SNA68*.

Therefore, it is somewhat surprising that it seems to have been ignored in the literature on chain volume measures, which has generally shown more interest in an arithmetic or a geometric mean of annual base prices. (See, for example, Diewert(1995)) Since much of this literature is at a fairly high level of abstraction it is not altogether clear exactly what annual base prices will be averaged. Presuming that these were calculated as in (2.1), since this is the only sensible way to calculate such annual prices, a geometric mean of the base prices for years 0 and 1 combined would be a rather odd hybrid, an unweighted geometric mean of weighted harmonic means:

$$\overline{p}_{01}^{w} = \sqrt{\overline{p}_0 \times \overline{p}_1} = \sqrt{1/\left(\sum_q w_{0q}(1/p_{0q})\right) \times \left(\sum_q w_{1q}(1/p_{1q})\right)} \qquad (4.4)$$

On the other hand, if they were calculated as unweighted geometric means throughout, then one would have:

$$\overline{p}_{01}^{w2} = \sqrt[8]{\prod_{y=0}^{1}\prod_q p_{yq}} \qquad (4.5)$$

which would only compound the problems that exist with the first estimate.

Formula (4.4) does not satisfy the property of transactions equality between the two years 0 and1, both years having about the same impact on the calculation of the average price, even if the volume of transactions were much greater in one year than the other. (This is a real possibility for new commodities, antedated commodities or highly cyclical commodities.) The unit values perfectly satisfy this requirement: at the limit, if all transactions were in year 0 and none in year 1, the unit value would be calculated for year 0 only.

Formula (4.5) is even worse, not satisfying the property of transactions equality between any pair of quarters as well as between the two years.

If the price of a commodity were zero in either year, using (4.4), or in any quarter, using (4.5), the base price would be zero, and the commodity would have no influence on the estimate of volume change for the year. If there are no sales of the commodity in a given year or quarter, one must impute a price for it anyway. It doesn’t simply disappear from the calculation as it does in calculating unit values.

Third, the ratio of the expenditures at current prices to the expenditures at constant prices provides an implicit price index:

$$P_{t/01} = \sum p_t q_t / \sum \overline{p}_{01} q_t$$

This is the Edgeworth-Marshall counterpart to the Paasche price index and would serve much the same purpose. Operationally, one can choose between calculating volume aggregates directly or by deflating by these implicit price indexes. However, like the Paasche price indexes, these Edgeworth-Marshall implicit price indexes would be poor indicators of price change in and of themselves, since all quarterly changes would be distorted by

Fourth, as with the Laspeyres volume aggregates these volume series would be additive across all important dimensions, and so would satisfy the property of matrix consistency. Components would sum to aggregates, quarterly values would sum to annual totals and regional estimates would sum to the national totals.

Fifth, the change between any two quarters or any two years of the Edgeworth-Marshall volume aggregates satisfies the identity test: if there is no change in any of quantities in the volume aggregate, then there will be no change in the volume aggregate. This is because, like the Laspeyres aggregates, all expenditures for all periods converted to the same set of fixed prices.

Sixth, the Edgeworth-Marshall volume aggregates are consistent in aggregation, that is, they can be equally well calculated from basic components or from higher level subaggregates.

Seventh, the Edgeworth-Marshall volume aggregates would be calculated as a chain volume series, whose base would change every year. This would also be true of the Edgeworth-Marshall volume aggregate. For the current year the estimate would be:

$$\sum \bar{p}_{t-1\&t} q_t \qquad (4.6)$$

while earlier years would follow this recursion:

$$\frac{\sum \bar{p}_{t-1\&t} q_t}{\sum \bar{p}_{t-2\&t-1} q_t} \times \sum \bar{p}_{t-2\&t-1} q_{t-1} \qquad (4.7)$$

That is, the volume aggregate would always be linked backwards, so that the additivity of the most recent periods would be preserved. This is different from what is recommended in *SNA93* where chaining is forward, and additivity is not preserved for the most recent volume estimates, even if a Laspeyres formula is used rather than a Fisher formula to calculate them. In an era of electronic publications there is no great difficulty in changing the base of volume estimates every year, as this would require. This has been proven by the Australian and British National Accounts which have adopted backward linking for their annually-linked chain volume measures, based on the Laspeyres formula. For the monthly GDP by industry estimates on which I work, we would normally publish from It is possible to have from 43 to 54 months of data that were additive over an annual production cycle with backward linking in place, due to delays in the updating of benchmark price estimates.

These estimates would be representative in a way that the old volume estimates were not, since the base prices would be updated every year. They would also satisfy the requirement of transactions equality, since every year y would be used to calculate the base prices for two annual comparisons: y-1 to y and y to y+1, and the importance of year y in the pooled set of base prices would depend on the volume of transactions associated with year y.

This is in contrast with the Fisher cross recommended by *SNA93*, where the principle of transactions equality is violated, each year's transactions having about the same influence on the measure of base prices whatever their volume. As can be seen from (4.1), if the volume of transactions in year 0 is very small compared to year 1, the base prices for years 0-1 will essentially be year 1 base prices. In the Fisher cross, by contrast, one would have:

$$Q_{1/0}^{F} = \left[\frac{\sum p_0 q_1}{\sum p_0 q_0} \frac{\sum p_1 q_1}{\sum p_1 q_0} \right]^{1/2} \qquad (4.8)$$

and prices for both years would have essentially the same influence on the measured volume movement even though the volume of transactions in year 0 are grossly inferior to those in year 1. Given a negative correlation between prices and quantities this would imply much higher measured rates of growth using the Fisher measure than using the Edgeworth-Marshall measure.

$$\left[\frac{\sum p_0 q_1}{\sum p_0 q_0} \frac{\sum \lambda p_1 q_1}{\sum \lambda p_1 q_0} \right]^{1/2} = \left[\frac{\sum p_0 q_1}{\sum p_0 q_0} \frac{\lambda \sum p_1 q_1}{\lambda \sum p_1 q_0} \right]^{1/2} = \left[\frac{\sum p_0 q_1}{\sum p_0 q_0} \frac{\sum p_1 q_1}{\sum p_1 q_0} \right]^{1/2}$$

Note also that another index number formula often associated with the Edgeworth-Marshall formula, the Bowley-Sidgwick formula, does not satisfy the transactions equality principle either. It is the arithmetic equivalent of the Fisher formula, defined as the arithmetic mean of the Laspeyres and Paasche indexes:

$$Q_{1/0}^{BS} = \left[\frac{\sum p_0 q_1}{\sum p_0 q_0} + \frac{\sum p_1 q_1}{\sum p_1 q_0} \right] / 2$$

Again, it can be seen that even if the volume of transactions is much larger in period 1 than in period 0, with this kind of a cross, prices of period 0 and period 1 have about the same impact on the index. (In fact, the Bowley-Sidgwick formula would give an even worse result in this respect than the Fisher formula. Since the geometric mean is always less than the arithmetic mean of two estimates, the higher estimate based on the period 0 prices would have a greater influence on the Bowley-Sidgwick estimate than on the Fisher estimate.)

Moreover, the Bowley-Sidgwick formula is also biased. It does not pass the time reversal test, that is:

$$Q_{0/1}^{BS} = \left[\frac{\sum p_1 q_0}{\sum p_1 q_1} + \frac{\sum p_0 q_0}{\sum p_0 q_1} \right] / 2 \neq 1 / Q_{1/0}^{BS} = 2 \Big/ \left[\frac{\sum p_0 q_1}{\sum p_0 q_0} + \frac{\sum p_1 q_1}{\sum p_1 q_0} \right]$$

and so, like the Laspeyres formula, although much less so, is subject to chain index drift.

5. Subannual Values of Annually Linked Chain Aggregates

Most countries publish GDP estimates at constant prices quarterly or monthly, and these subannual estimates are of at least as great interest to users as the annual estimates. Unfortunately, papers on chain volume aggregates written by interested economists or even by official statisticians have tended to ignore the difficulties in creating quarterly series with annual links. Most papers on the subject deal with the characteristics of different formulas, Fisher, Montgomery-Vartia and so forth, for individual, read annual, links and one is left with the impression that these same properties of the annual links also exist for the quarterly growth ratios. As a general rule, this is not so, and some formulas which look very good in terms of annual links, look much worse if one considers what kind of quarterly estimates they will generate.

One of the few works that does recognize this problem is chapter XVI of *SNA93*. Unfortunately, its conclusion, that "chain indices should only be used to measure year-to-year movements and not quarter to quarter movements" is unduly defeatist.[8] This conclusion seems to be based on its observation "that if it is desired to measure the change in prices or volumes between a given month, or quarter, and the same month, or quarter, in the following year, the change should be measured directly and not through a chain index linking the data over all the intervening months, or quarters." (See *SNA93*, section 16.49.)

However, annually linked volume aggregates whose constant price structure changes every year are not much different from the old *SNA68* volume measures. The growth rates measured between consecutive quarters would all be comparable (i.e. they would satisfy the proportionality test) except for the first quarter. Over a 15-year span covering 59 quarterly changes, the *SNA68* aggregates would provide 57 comparable quarterly changes,or 95% of the total (the first quarter changes for the years following link years being non-comparable).

[8] This conclusion seems to be based on his observation "that if it is desired to measure the change in prices or volumes between a given month, or quarter, and the same month, or quarter, in the following year, the change should be measured directly and not through a chain index linking the data over all the intervening months, or quarters." (See *SNA93*, section 16.49.) But annually

first quarter changes for the years following link years being non-comparable). Chain volume aggregates would provide 45 comparable quarterly changes, or 75% of the total. For monthly GDP series, only January monthly changes would be non-comparable, so comparable monthly changes would be 91.7% of the total.

And for that matter, if Edgeworth-Marshall volume aggregates were calculated, the only distortion in a fourth quarter to first quarter comparison would be in the replacement of one set of base prices by another, both sets very current, and both sharing half of their prices in common. In many cases the impact of the change in price structure would be negligible. This would also be true of four-quarter comparisons, which would be based on a single link and not on three links as Hill postulates.

Users of SNA data want to have one official set of quarterly and annual estimates, and not go to one series for annual growth rates, and another, possibly with quite different annual movements, for quarterly growth rates. However, it is certainly true that users should be accommodated by providing overlapping sets of direct volume aggregates that provide meaningful quarterly growth rates where the chain volume aggregates do not do so. In many cases, these alternative series would likely only provide assurance that the quarterly growth rates generated by the chain volume aggregates were not very different from those provided by measures of pure volume change.

The quarterly volume index estimates equivalent to the Fisher annual estimates would have the following formula:

$$Q^F_{1q/0} = \left[Q^L_{1q/0} \times Q^P_{1q/0}\right]^{1/2} = \left[\frac{\sum p_0 q_{1q}}{\sum p_0 q_0}\frac{\sum p_{1q} q_{1q}}{\sum p_{1q} q_0}\right]^{1/2} \qquad (5.1)$$

This is the same formula as (4.8) except that all period 1 quantities and the Paasche index's period 1 prices, are now for a specific quarter rather than for a year. As far as this particular index comparison goes, proportionality is still satisfied, since what we have is still have the geometric mean of two fixed-price volume indexes. Factor reversal is also satisfied, since:

$$Q^F_{1q/0} \times P^F_{1q/0} = \left[\frac{\sum p_0 q_{1q}}{\sum p_0 q_0}\frac{\sum p_{1q} q_{1q}}{\sum p_{1q} q_0}\right]^{1/2}\left[\frac{\sum p_{1q} q_0}{\sum p_0 q_0}\frac{\sum p_{1q} q_{1q}}{\sum p_0 q_{1q}}\right]^{1/2} = \frac{\sum p_{1q} q_{1q}}{\sum p_0 q_0}$$

However, since there is also annual rather than quarterly linking, the quarterly growth ratios of this index do not satisfy proportionality:

$$Q^F_{1q/0} \,/\, Q^F_{1q-1/0} = \left[\frac{\sum p_0 q_{1q}}{\sum p_0 q_0}\frac{\sum p_{1q} q_{1q}}{\sum p_{1q} q_0}\right]^{1/2} \Big/ \left[\frac{\sum p_0 q_{1q-1}}{\sum p_0 q_0}\frac{\sum p_{1q-1} q_{1q-1}}{\sum p_{1q-1} q_0}\right]^{1/2} = \left[\frac{\sum p_0 q_{1q}}{\sum p_0 q_{1q-1}}\frac{\sum p_{1q} q_{1q}}{\sum p_{1q} q_0} \Big/ \frac{\sum p_{1q-1} q_{1q-1}}{\sum p_{1q-1} q_0}\right]^{1/2}$$

since even in the absence of price change by any commodity, the growth rate will generally be different from one due to price movement sbetween the base year and the quarters of the current year. The problem stems from the Paasche component of the Fisher index, and not from the Laspeyres component; a direct Paasche index, because its relative price structure changes every period, doesn't satisfy the proportionality axiom for multi-period comparisons; therefore, neither does the direct Fisher index.

Formula (5.1) can be simplified by replacing its Paasche component with a fixed-price index reflecting the price structure of year 1:

$$Q^{GMFP}_{1q/0} = \left[Q^L_{1q/0} \times Q^1_{1q/0}\right]^{1/2} = \left[Q^0_{1q/0} \times Q^1_{1q/0}\right]^{1/2} = \left[\frac{\sum p_0 q_{1q}}{\sum p_0 q_0}\frac{\sum p_1 q_{1q}}{\sum p_1 q_0}\right]^{1/2} \qquad (5.2)$$

where the superscript GMFP indicates a geometric mean of fixed-price volume indexes. Following the second equality sign, the superscripts of the volume indexes indicate what year they take their fixed price structure from, $Q^0_{1q/0}$ is thus identical with $Q^L_{1q/0}$, the Laspeyres index. In some cases, in the absence of quarterly price data, such a formula might be used to represent a Fisher volume index, although strictly speaking it isn't one. A GMFP index does satisfy proportionality for its quarterly growth rates:

$$Q^{GMFP}_{1q/0} \,/\, Q^{GMFP}_{1q-1/0} = \left[\frac{\sum p_0 q_q}{\sum p_0 q_0}\frac{\sum p_1 q_{1q}}{\sum p_1 q_0}\right]^{1/2} \Big/ \left[\frac{\sum p_0 q_{1q-1}}{\sum p_0 q_0}\frac{\sum p_1 q_{1q-1}}{\sum p_1 q_0}\right]^{1/2} = \left[\frac{\sum p_0 q_{1q}}{\sum p_0 q_{1q-1}}\frac{\sum p_1 q_{1q}}{\sum p_1 q_{1t-1}}\right]^{1/2}$$

as one would expect for the geometric mean of the growth rates of two fixed-price volume indexes. However, the GMFP index does not satisfy the factor reversal test when multiplied by its equivalent geometric mean fixed basket (GMFB) index:

$$Q^{GMFP}_{1q/0} \times P^{GMFB}_{1q/0} = \left[\frac{\sum p_0 q_{1q}}{\sum p_0 q_0}\frac{\sum p_1 q_{1q}}{\sum p_1 q_0}\right]^{1/2} \left[\frac{\sum p_{1q} q_0}{\sum p_0 q_0}\frac{\sum p_{1q} q_1}{\sum p_0 q_1}\right]^{1/2} \neq \frac{\sum p_{1q} q_{1q}}{\sum p_0 q_0}$$

These geometric mean indexes are the quarterly indexes that are closest to Fisher indexes while retaining the proportionality property. Rather than leaving it up to the different national statistical agencies what to do, this is what SNA 1993 should have prescribed for quarterly measures when it advised the adoption of the Fisher formula for annual measures. However, these measures have not so far been adopted by either the United States or Canada, the only countries that have so far adopted the Fishe formula for their chain volume measures. individual countiresofficial statisticians what to do, this is what SNA 1993

Often there is a greater level of detail available for both price indexes and value series at the annual level than at the quarterly level so there is a need to adjust a quarterly volume indicator to annual benchmarks. For a geometric mean volume index it would be inappropriate to directly adjust it so that it averaged to a Fisher volume index. Instead one would want to benchmark the quarterly chain Laspeyres volume index to its annual benchmarks, the quarterly chain fixed-basket index compatible with a

Much has been made of the fact that though the Fisher volume aggregates are not additive one can precisely calculate component contributions to changes in the aggregate index, just as one can for a constant-price volume aggregate. However, this decomposition is made possible because the Fisher formula passes the strong factor reversal test, so no such decomposition is possible for the geometric mean index shown in (5.2). One could still calculate the impact of changes in individual components on the percent change of the total index, but these would not add to the aggregate percent change as component contributions to change should.

Since the additivity problem is usually discussed in a general sense, many people may be unaware of the crucial distinction between additivity of quarters to years and other types of additivity (over commodities, industries or regions) in calculating annually-linked chain volume indexes. Generally speaking, chaining destroys all types of additivity except the addditivity of quarterly values to annuals, where that additivity exists to begin with. Therefore it is simply not true that there is not much difference, as far as additivity is concerned, between chain fixed-price volume measures such as the Edgeworth-Marshall series and other measures, whether Fisher, Sato-Vartia or whatever that do not have this additivity property. In each and every year of a chain fixed-price volume series, the quarterly index numbers will average to their annual values; for other chain volume measures they won't.

Unfortunately, the implementation of chain Fisher volume measures by both the U.S. BEA and the CSNA has muddied these distinctions. Both have imposed additivity on their quarterly chain Fisher measures, although it is illogical to do so. The BEA calculates quarterly-linked chain Fisher indexes and benchmarks them to annually-linked chain Fisher benchmarks. The CSNA has also calculated quarterly-linked chain Fisher indexes, but has not benchmarked these estimates to anything at all; the annual estimate is the arithmetic mean of the quarterly estimates.

Formula (5.1) is a better way to decompose an annually-linked index than using a quarterly-linked distributor, if deflation occurs at the same level of detail at the annual level and the quarterly level. If this is not the case, the appropriate thing to do would be to adjust a quarterly distributor of chain Laspeyres annual benchmarks and another distributor to chain Paasche annual benchmarks, then month by month calculate the geometric mean of the two series. (Assuming that it were appropriate to have a quarterly-linked chain Fisher distributor, it should be adjusted to the chain Fisher benchmarks by using the chain Laspeyres quarterly series as the distributor for the chain Laspeyres benchmarks and the chain Paasche quarterly series for the chain Paasche benchmarks.) Whatever distributors were used, the quarterly index numbers would not average to the annual benchmarks.

As for the CSNA measure, given that the quarterly index numbers are based on quite different price structures their arithmetic average is essentially meaningless. It would have been much more appropriate to calculate their annuals as a geometric mean since the chain series is nothing but a strand of quarterly growth ratios linked together.

Both agencies seem to have had misgivings about the consequences that adopting the Fisher formula would have for the additivity of their estimates, and this has kept both of them from calculating annuals and quarterlies in a way that is logical and consistent. This is especially surprising in the BEA because they have been publicly dismissive of the value of additivity in national accounting (see, for example, the 2000 paper by Ehemann et al.).

Why doesn't the BEA practice what it preaches? Or, to turn the question around, why doesn't it preach what it practices. Given that it is so unwilling to give up additivity of quarterly estimates to annuals that it will impose it, against all logic, on its chain Fisher estimates, why doesn't the BEA advocate and implement chain fixed-price volume aggregates like the Edgeworth-Marshall measures that would legitimize such additivity? And the same goes for the CSNA.

The use of quarterly-linked chain estimates by both the United States has a serious potential for degrading the quality of growth estimates so it merits further comment. As Hill remarks in *SNA93*:

> a chain index should be used when the relative prices in the first and last periods are very different from each other and chaining involves linking through intervening periods in which the relative prices and quantities are intermediate between those in the first and last periods.[9]

If, for example, one wants to know the difference in output between the first quarter of 2001 and the first quarter of 2003 linking seven times through different quarters will hardly give us in each and every case a relative price structure that departs smoothly from that of the initial

[9] See *SNA93*, p.388.

quarter in the direction of the terminal relative prices in the next quarter will inevitably be intermediate between those of the first and last quarters. Suppose one were measuring gross output of the air transport sector. Does it really make sense to link through the prices of third quarter and fourth quarter 2001, given the highly abnormal pricing situation in the wake of the September 11th atrocities? The consequences of linking where one shouldn't be are reduced because the Fisher formula has the time reversal property, but this in no way justifies linking inappropriately.

Some commodities are seasonally disappearing and this is especially a problem for northern countries like Canada. Prices may well be missing for one or both quarters of a quarterly comparison; more seriously if quantities go to zero, one must calculate output relatives with a zero base, which makes the index number undefined.

This problem is only partially resolved by the seasonal adjustment of economic time series. First, even if a statistical agency only publishes GDP at constant prices as a seasonally adjusted series, it is sound statistical practice to calculate the same aggregate unadjusted for seasonal variation. Any chain linking procedure that makes the calculation of such raw estimates impossible, or requires so much tinkering to calculate them that their comparability with the official seasonal adjusted series is compromised, has little to recommend it.

Second, one is necessarily resorting to highly artificial base prices. Certainly seasonal adjustment procedures can arrive at a first quarter price for field-grown corn one way or another, but it is not and can never be a solid number, as is its annual unit price.

Although both BEA and CSNA procedures involve quarterly linking, the BEA procedures are better, because at least the quarterly-linked estimates are adjusted to good annual benchmarks. The Canadian estimates are not, which leaves open the possibility of substantial chain index drift in one direction or another.

Another danger with the CSNA procedure is that more than anything else, the quality of output estimates at constant prices depends on the level of disaggregation at which the estimattion takes place. Until one gets down to a level more appropriate to sampling than to aggregation, the more disaggregate the estimates the better.[10] More disaggregated information on prices, revenues or production volumes is, and always will be available at the annual than at the quarterly level. By creating a methodology that requires quarterly estimates, the CSNA encourages the calculation of production estimates to occur at too gross a level of detail.

[10] On this topic, see the paper by Horner.

One of the properties of the Fisher formula is proportionality, i.e. if all quantities of all commodities increase by the same percentage, that will be the rate of increase of the Fisher volume index. Note that the geometric mean index shown in (5.1) also has the proportionality property for quarterly changes.. On the other hand, the US BEA measures only approximately have it for quarterly changes. The quarterly changes of their quarterly-linked series would have this property prior to benchmark adjustment, but not afterwards. In other words, the quarterly data may show no quantity change from one quarter to another in any commodity, but due to benchmark adjustment an increase or decrease in output will nonetheless be indicated. The BEA documents on their chain measures have not paid any attention to this important caveat.

Nonetheless, the BEA measures do satisfy proportionality for annual changes. This is not true of the CSNA measures. Calculated as the average of the quarterly Fisher estimates, different years are not compared based on the same sets of relative prices, so the annual estimates could should an increase or a decrease even if all outputs were unchanged.

In other words, the BEA has chosen to give priority to their annual growth rates, the CSNA to their quarterly growth rates. Obviously, the BEA has made the better choice; the quality of quarterly estimates will never be up to the standard of the annual estimates, and anyone familiar with the compilation of quarterly data must be aware of many factors that make measured quarterly changes not strictly comparable despite the best efforts of their compilers.

However else the CSNA changes its methodology for calculating chain volume estimates, it really should switch to chaining annually using whatever formula. Chaining quarterly with no annual benchmarks is contrary to *SNA93*, contrary to BEA practice, contrary to Eurostat practice and contrary to common sense.

Quarterly- or monthly-linked indexes do have their uses. For example, they are really a required adjunct if one is calculating seasonal-basket price indexes and wishes to make some sense of quarterly or monthly price movements. But they are basically only useful as adjuncts to official series, not as official series in themselves.

Besides the Fisher formula, a number of log change formulas have been recommended for measuring volume indexes, including the Törnqvist, Montgomery-Vartia formula and the Vartia-Sato formula. All conform to the general formula:

$$Q_{1/0}^{LC} = \prod \left(q_1 / q_0\right)^{w} = \sum w(\ln(q_1) - \ln(q_0))$$

whence the name log-change indexes.

The Törnqvist index is mentioned as an alternative to the Fisher index in *SNA93*. The Montgomery-Vartia formula was recommended as the best technical formula by consultants to Eurostat for its good properties, despite the fact that it does not meet the proportionality test. By the way, this report dismissed the Fisher formula from consideration because it is not consistent in aggregation.[11] The closely related Sato-Vartia formula has also attracted considerable interest. Like the Fisher formula it passes both the factor reversal and the time reversal test, and it is exact with respect to a CES utility function.[12]

However, the Törnqvist formula has a serious problem when production goes to zero for a commodity. Technically the index becomes undefined, since the logarithm of zero does not exist. This is a real problem for calculating quarterly and especially monthly GDP estimates since production of some commodities will go to zero for seasonal or other reasons. Vartia states in his paper that the Sato-Vartia formula will handle quantities or prices going to zero gracefully. However, this is only true if the index weights and component index numbers are changing at the same time. If one is linking annually but calculating quarterly or monthly index estimates, and output does not go to zero in all quarters of the year, the Sato-Vartia index will also be undefined. For calculating subannual series then, it is hardly any more robust than the Törnqvist formula, since it is much more likely that production will drop to zero for a few months than for an entire year.

Incidentally, these practical problems with the Sato-Vartia formula make one wonder how seriously one should take the requirement that an index number formula be superlative, that it be exact for a given utility function. After all, the Sato-Vartia index is exact for the CES utility function, one of the most frequently used functions in econometric work because of its useful properties. But it becomes undefined for situations that occur in calculating quarterly economic accounts all of the time. Does it really make any sense then, to dismiss the Edgeworth-Marshall formula from consideration because it is not exact for any utility function?

6. Analysis of Changes for Quarterly Chain Volume Aggregates

Any annually-linked chain volume aggregate will have a problem of non-comparability for quarterly changes for the first quarter, due to the switch from one relative price structure to another. However, it is possible to precisely measure the distortion due to this switch. The quarterly percent change in the volume aggregate if there were no change in the relative price structure would be:

[11] See Al et al, p.354 for dismissal of the Fisher formula and p.355-56 for the endorsement of the Montgomery-Vartia formula, what they call the Vartia-I formula.

[12] See the articles by Vartia and by Reinsdorf and Dorfman for more on this fascinating formula.

$$100\times\sum \bar{p}_{t-2\&t-1}(q_{t,1}-q_{t-1,4})/\sum \bar{p}_{t-2\&t-1}q_{t-1,4} \qquad (6.1)$$

The difference between this quarterly percent change and the actual change in the official index is the measure of the interaction between the change in the relative price structure and output change. In most cases, this difference would not be great. Annual prices are less volatile than quarterly prices to begin with, and a two-year moving average of annual prices would do much to iron out the fluctuations in annual movements. However, where the difference was considerable for an official aggregate a publication release could draw attention to the pure volume change measure, and deemphasize the official quarterly estimate of volume change.

The same principle would apply in the case of four-quarter percent changes. The percent change in the chain volume aggregate from the fourth quarter of year t-1 to the fourth quarter of year t if there were no change in the relative price structure would be:

$$100\times\sum \bar{p}_{t-2\&t-1}(q_{t,4}-q_{t-1,4})/\sum \bar{p}_{t-2\&t-1}q_{t-1,4} \qquad (6.2)$$

and the difference between this percent change and the official estimate would show the distortion of the measured growth rates due to annual linking.

It should be noted that if the Edgeworth-Marshall aggregates were linked at the fourth quarter and not at the year, then the four-quarter percent change of the official estimates would be a measure of pure volume change:

$$100\times\sum \bar{p}_{t-1\&t}(q_{t,4}-q_{t-1,4})/\sum \bar{p}_{t-1\&t}q_{t-1,4} \qquad (6.3)$$

and there would be no distortion due to changes in relative price structure. However in this case the annual price movements would be compromised.

In Statistics Canada, linking at the fourth quarter is the practice for some, but not all price indexes whose baskets are updated once a year.[13] Regardless of whether this is the best practice for those price indexes, it should be observed that the context is different here. There are no annual benchmarks for the price indexes that are linked at the fourth quarter. Their annual data is derived from their quarterly data. For SNA volume aggregates, there are annual benchmark estimates for many series and in some cases sub-annual estimates are non-existent, linking at the fourth quarter would make no sense, and the only sensible choice is to link at the year.

A couple of recent papers by American economists who work outside of the BEA have outlined quite arcane calculations to derive the contributions of components to growth for the US chained volume measures.[14]Both provide quite complicated calculations that would likely be beyond the sophistication of most users of SNA data and neither seems to give sufficient attention to the obvious fact that when comparing chained volume estimates across link periods strictly speaking, no precise calculation of contribution shares to growth is possible.

For longer time periods, say the segment 1991Q1 to 1998Q1 that Rossiter considers in his paper, output comparisons based on a single set of constant prices is at least dubious. The best that can be done is to look at the contributions of each component to total GDP growth based on published chain estimates. This is why national accountants should continue to publish actual values for volume aggregates and not just volume indexes.[15] Over shorter time periods, say the most recent four to six years, users can and should be provided with fixed-price volume aggregates that will allow them to calculate component contributions to growth in a precise way.

Such fixed-price series would also be useful for the dating of business cycles. This is a delicate task at best, and it would become basically impossible if one had only chained volume estimates to work. And they would be useful to monitor the changes in volume shares of components of GDP from one period to another where chain volume estimates are non-additive. (As discussed above, for the most recent year or years, the Edgeworth-Marshall estimates would be additive, so one could compare say the share of ICT products in current quarter GDP with the share in the previous quarter cleanly and correctly.) In his paper, Whelan acknowledges that chain measures are not particularly useful for this task and argues instead that one should base comparisons of component shares on expenditures at current prices, ignoring the inflationary distortions inevitable in such a procedure. This is simply ridiculous, and would mean a big step backward for analysis of economic statistics if it ever became commonplace.

In principle, it would be possible to calculate official chain Fisher volume estimates and direct fixed-price volume estimates at the same time. In fact, this is what *SNA93* recommended, supplementing annual chain Fisher volume indexes with Laspeyres volume indexes. But it is difficult to see why one would want to create one set of direct Fisher volume series for chaining and another set of direct Laspeyres volume series for analysis. It would be much more

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[13] The non-residential building construction price index and the apartment building construction price index are linked at the fourth quarter; the air fares index is linked at the year.

[14] See the articles by Rossiter (p.369-70) and Whelan (p.8-10).

[15] Of course, over such long spans, the measured contributions to change would be somewhat sensitive to what period was the base for the volume aggregates at constant prices, and the current year may not always be an

sensible to create a single set of fixed-price volume series and chain these together. Direct Laspeyres volume series would not be the best fixed-price series to use, since they are upward biased. It would be better to use a fixed-price formula that satisfies the time reversal test, which essentially means the Edgeworth-Marshall or the Walsh formula. Either of these would be acceptable, but the Edgeworth-Marshall formula is superior because it passes the transactions equality test.

7. An Aside on Price Indexes for GDP

The implicit price indexes for GDP that are derived from the Edgeworth-Marshall volume aggregates would not be the most suitable price measures for the components of GDP. In order to be consistent with the calculation of the volume measures, it would make to sense to also calculate annually-linked chain Edgeworth-Marshall price indexes. Each link of these indexes would have the formula:

$$P_{1é0}^{EM} = (\sum p_1(q_0 + q_1)/2) / \sum p_0(q_0 + q_1)/2 = (\sum p_1(q_0 + q_1)) / \sum p_0(q_0 + q_1)$$

Note that the calculation of average quantities over the two years for the Edgeworth-Marshall price index is based on a simple mean whereas the calculation of average prices over the two years is based on a weighted harmonic mean. There is no inconsistency here. As can be seen there is no difference in terms of result between weighting by the mean of quantities or the sum of quantities. The most natural way of combining the two periods quantities is simply to add them together, just as the most natural way of calculating their average price is to take their unit values. It is difficult to see how anyone can argue that it is more natural to take the geometric mean of quantities in years 0 and 1, i.e. to calculate a Walsh price index, than to take their sum. Moreover, the Walsh index will simply ignore any commodity that is available in only one of the two periods, the since the geometric mean of a positive value and a zero value is zero. (Although arguably this is an advantage if one would like to reduce the scope of imputations in the SNA estimates.)

In some cases, for commodities with highly volatile production profiles, a two-year basket may not be adequate to generate a stable weighting pattern.[16] For such commodity groups or industries, the chain Edgeworth-Marshall series could be replaced with similar indexes based

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appropriate choice for the base year. I would think there is a strong argument in the case of looking at growth between two distant quarters to use the earlier quarter as the base.

[16] Within Statistics Canada, the new housing price index, non-residential building construction price index and apartment building construction price index all have three-year baskets; the farm product price index has a five-year basket.

on three- to five-year baskets. At higher levels of aggregation, the chain Edgeworth-Marshall formula would continue to be employed.

Such an adaptation of the Edgeworth-Marshall formula would not pass the time reversal test as defined above. But that is not such a big problem; it would still break the close link between basket change and price change that makes the use of the Laspeyres formula a hazard in chain computations. Unlike the chain Laspeyres series, they would not be very mcuh subject to chain index drift.

8, Conclusion

The recommendations of this paper regarding chain indexes are fairly conventional as regards linking. I agree with the idea of chaining at the annual level and that is also what the *SNA93* recommends and what most countries that calculate experimental or official chain volume series have done.

I also support the calculation of fixed-price volume aggregates rather than Fisher volume indexes. This conclusion, although at variance with *SNA93*, the BEA and the CSNA, is in keeping with the decisions made by Eurostat and by the Australian Bureau of Statistics, and by a number of economists who have studied the subject.

However, most economists who have favoured chaining using fixed-price volume series have opted for Laspeyres aggregates, with only Erwin Diewert, so far as I know, favouring the Walsh formula. No-one, to the best of my knowledge, has ever recommended the Edgeworth-Marshall formula for use in the SNA. Statistics Sweden planned to revamp their consumer price index as a chain Edgeworth-Marshall index but finally they opted for the quite similar Walsh formula instead.

In recent years, the German economist Claude Hillinger has recommended using Edgeworth-Marshall price indexes as deflators to calculate volume aggregates.[17] Contrary to normal practice, the deflator for any commodity group would not be representative of that group but would be an Edgeworth-Marshall price index representing a higher level aggregate. It is not within the scope of this paper to comment on Professor Hillinger's work in detail, but it should be underlined that his proposal is quite different from the one in this paper. However, Professor Hillinger did do a good thing in bringing renewed interest to the Edgeworth-Marshall formula.

[17] See Hillinger (2000) and the papers that comment on his methdology at the same OECD meeting (not listed in the references, except for Ehemann et al.(2000).

This formula has always had its defenders, from Knibbs in the 1920's to Krtscha in the contemporary period, but it has never had as strong backing as other formulas. In my view this is due to an unnatural emphasis in choosing index formulas on whether they pass the factor reversal test or whether they are exact for a utility function. Time reversal, matrix consistency (or additivity), the new good property and the property of transactions equality are surely more important considerations in choosing a chain index formula, and the Edgeworth-Marshall formula is unique in possessing all of these properties.

Possibly it has been ignored because the Walsh formula, which is exact for a utility function, is just as good with respect to time reversal, matrix consistency and the new good property and only falls down slightly on the principle of transactions equality.

If I had only a sixty-second TV slot to deliver my message, this would be it:

Chain price and volume aggregates with annual links hold the promise of improved measures of growth. So it is most unfortunate that discussion has centred on two formulas, the Laspeyres and the Fisher, neither of which is well-suited to the calculation of chain aggregates. The Laspeyres formula doesn't pass the time reversal test, and a healthy fear of chain drift should lead us to reject it. The Fisher formula does pass the time reversal test, but it fails the matrix consistency test, is not even weakly consistent in aggregation and fails the new good test, all of which make it an awkward and unsatisfying formula for both producers and consumers of National Accounts estimates. None of these criticisms holds for the Edgeworth-Marshall formula and only the Edgeworth-Marshall formula has the important property of tranasctions equality, so important in its implications for growth measurement in the new economy. So far as any one formula will be the formula for price and volume measurement in the National Accounts in the 21st century, it should be the Edgeworth-Marshall formula. However, we should cease to try to make all industries or commodities fit to the Procrustean bed of one formula and where required we should change our formulas for price and volume measures to take account of cyclical commodities and for price measures to take account of seasonal or irregular commodities. Finally, matrix consistency is dependent not only on formula choice but on linking policy. We must follow the good example of Australia and Britain, and change our reference year annually, linking our chain-volume estimates backward rather than forward.

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