An axiomatic approach to the measurement of productivity

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Abstract

This paper shows how both the presence and extent of productivity changes may be inferred nonparametrically from panel data on a firm's outputs, it's inputs and the prices it pays for inputs. The approach uses the nonparametric characteristisation of cost minimising behaviour developed by Afriat (1972), Hanoch and Rothchild (1972), Diewert and Parkan (1983) and Varian (1984) and an assumption on the direction of productivity improvement over time to provide the necessary empirical restrictions. We show how these two assumptions are empirically testable, and illustrate the method using panel data on a sample of Danish farms.

1 Introduction

In this paper we ask whether the evolution of a firm's productivity may be nonparametrically inferred from data on its inputs, outputs and their corresponding prices over time. We show that by making two identifying assumptions (one on firm behaviour and the other on the direction of productivity change) this is indeed possible. We also show how these joint identifying assumptions are empirically rejectable.

The approach in this paper is based on the nonparametric approaches to production analysis[1] and efficiency measurement[2] The typical appraoch in this literature is to measure differences in relative efficiency between firms in a cross section assuming the same basic technology across units[3]. Our approach is similar to these studies, however, we use panel data and allow for heterogenous technology across firms and for heterogeneous rates of technological improvements over time. The plan of the paper is as follows. The next section discusses the theoretical issues. Section 3 presents some empirical illustrations of the idea based on panel data on a sample of Danish farms. Section 4 concludes.

[1] Afriat (1972), Hanoch and Rothchild (1972), Diewert and Parkan (1983) and Varian (1984)

[2] Farrell (1957), Afriat (1972) and Diewert and Mendoza (1996)).

[3] Given the same production technology the maximum production level may not be achieved by some firms due to information deficiencies, adjustment costs, lags etc.

2 Theory

In what follows let $\mathbf{q}$ denote a $J-$vector recording the quantities of the J outputs of a firm and let $\mathbf{x}$ be a $I-$vector recording the quantities of the I inputs. We will summarise the firm's technology by means of its inputs requirement set $V(\mathbf{q})$. This consists of all input vectors $\mathbf{x}$ that can produce at least the output vector $\mathbf{q}$ (in all J dimensions).

$$V(\mathbf{q}) = \{\mathbf{x} : \mathbf{x} \text{ can produce at least } \mathbf{q}\}$$

The input requirement set has the following properties

1. Monotonicity: *If* $\mathbf{x}$ *is in* $V(\mathbf{q})$ *and* $\mathbf{x}' \geq \mathbf{x}$, *then* $\mathbf{x}'$ *is in* $V(\mathbf{q})$.
2. Nestedness: *If* $\mathbf{x}$ *is in* $V(\mathbf{q})$ *and* $\mathbf{q} \geq \mathbf{q}'$, *then* $\mathbf{x}$ *is in* $V(\mathbf{q}')$.
3. Convexity: *If* $\mathbf{x} \in V(\mathbf{q})$ *and* $\mathbf{x}' \in V(\mathbf{q})$, *then* $\lambda\mathbf{x} + (1-\lambda)\mathbf{x}' \in V(\mathbf{q}), \lambda \in [0,1]$.
4. Regularity: $V(\mathbf{q})$ *is a closed non-empty set for all* $\mathbf{q}$.

The most general characterisation of technological change is in terms of changes in the input requirement set. In particular, productivity improvement can be viewed as the expansion of the input requirement set (since lower amounts of inputs can now produce $\mathbf{y}$). Consider (without loss of generality) two periods 0 and t. In period 0 the input requirement set is

$$V_0(\mathbf{q}) = \{\mathbf{x} : \mathbf{x} \text{ can produce at least } \mathbf{q}\}$$

In period t it is

$$V_t(\mathbf{q}) = \{\mathbf{x} : \mathbf{x} \text{ can produce at least } \mathbf{q}\}$$

We then have the following definition:

Definition. *The following statements are equivalent:*

(1) period t technology is more productive than period 0.

(2) $V_0(\mathbf{q}) \subseteq V_t(\mathbf{q})$.

In other words there are elements (input vectors) in $V_t(\mathbf{q})$ which previously couldn't produce the given level of outputs $\mathbf{q}$. Productivity and technological changes are therefore equivalent to changes in the firm's input requirement set over time and the measurement of productivity change is essentially a matter of comparing the set of inputs capable of producing at least $\mathbf{q}_t$ using period t technology $(V_t(\mathbf{q}_t))$ with the set of inputs capable of producing at least $\mathbf{q}_t$ using period 0 (reference) technology $(V_0(\mathbf{q}_t))$. There are many ways in which the relationship between the two sets might be measured. In this paper we concentrate on the approach discussed in Caves, Christensen and Diewert (1982) based on Malmquist quantity indices:

$$\frac{D_t(V_0, \mathbf{x}_t)}{D_t(V_t, \mathbf{x}_t)} = \frac{\min_{d_t}\{d_t : d_t\mathbf{x}_t \in V_0(\mathbf{q}_t), d_t > 0\}}{\min_{d_t}\{d_t : d_t\mathbf{x}_t \in V_t(\mathbf{q}_t), d_t > 0\}}$$

for $t = 0, 1, ...T$. The numerator $D_t(V_0, \mathbf{x}_t)$ is the distance function which measures the minimum radial distance between the observed $\mathbf{x}_t$ and the input requirement set $V_0(\mathbf{q}_t)$. This shows how much the period t inputs have to be scaled up in order to continue to produce period t outputs, but with reference period technology. The denominator measures how much period t inputs have to be scaled up in order to continue to produce period t outputs, period t technology. Clearly $D_t(V_t, \mathbf{x}_t) = 1$ and we need therefore only concentrate on the numerator. If we knew the properties and form of the $V_0(\mathbf{q})$ family of input requirement sets, then it would be a simple matter to apply this idea to the data. However, in this paper we ask what we can do without parametric knowledge of $V_0(\mathbf{q})$.

Suppose that the firm is cost-minimising but that its technology remained fixed at the reference state. Then the data should solve the problem

$$\min_{\mathbf{x}} \ \mathbf{w}_t'\mathbf{x} \text{ subject to } \mathbf{x} \text{ is in } V_0(\mathbf{q}_t)$$

for every period $t = 0, 1, ..., T$. The observable consequences of this are summarised in the following theorem.

> **Theorem 1**: (Hanoch and Rothschild (1972), Diewert and Parkan (1983), Varian (1984)). *The following conditions are equivalent:*
> *(1) there exists a family of nontrivial, closed, convex, positive monotonic input requirement sets* $\{V_0(\mathbf{q})\}$ *such that the data* $\{\mathbf{q}_t, \mathbf{w}_t, \mathbf{x}_t\}$ *solves the problem* $\min_{\mathbf{x}} \ \mathbf{w}_t'\mathbf{x}$ subject to $\mathbf{x}$ is in $V_0(\mathbf{q}_t)$ for each $t = 0, 1, ..., T$.
> *(2) if* $\mathbf{q}_j \geq \mathbf{q}_i$ *then* $\mathbf{w}_i'\mathbf{x}_j \geq \mathbf{w}_i'\mathbf{x}_i$ *for all* i *and* j.

The empirical condition in statement (2) is known as the Weak Axiom of Cost Minimisation (WACM), if this condition holds then there is an intertemporally stable family of input requirement sets which rationalise the data (in the sense set out in statement (1) of the Theorem). Conversely, if there is an intertemporally stable family of inputs requirement sets then the data must satisfy WACM. Now consider the case in which technology is improving and make the following two assumptions:

A1. the firm is cost-minimising.
A2. input productivity improvement occurs over time.

Under these assumptions then the data will solve the problem

$$\min_{\mathbf{x}} \ \mathbf{w}_t'\mathbf{x} \text{ subject to } \mathbf{x} \text{ is in } V_t(\mathbf{q}_t)$$

where

$$V_{t-1}(\mathbf{q}) \subseteq V_t(\mathbf{q})$$

for each $t = 1, ..., T$; but since the input requirement set is changing, then such a situation would be a *prima facie* cause of violations of WACM. We now consider two possible types of violation. First suppose that we observe

$$\begin{array}{rcl}\mathbf{q}_0 & \leq & \mathbf{q}_t \\ \mathbf{w}_0'\mathbf{x}_0 & > & \mathbf{w}_0'\mathbf{x}_t\end{array}$$

We then have the following result which relates this type of observation to our economic assumptions.

> **Lemma 1**: *The observations* $\mathbf{q}_0 \leq \mathbf{q}_t, \mathbf{w}_0'\mathbf{x}_0 > \mathbf{w}_0'\mathbf{x}_t$ *and assumptions A1 and A2 imply that* $\mathbf{x}_t \in V_t(\mathbf{q}_0)$ *and* $\mathbf{x}_t \notin V_0(\mathbf{q}_0)$.
>
> **Proof.** Nestedness implies that since $\mathbf{x}_t \in V_t(\mathbf{q}_t)$ and $\mathbf{q}_t \geq \mathbf{q}_0$, then $\mathbf{x}_t \in V_t(\mathbf{q}_0)$. Cost minimisation implies that $\mathbf{x}_t \notin V_0(\mathbf{q}_0)$.■

By Lemma 1 we know, given our assumptions and this type of WACM violation, that in period 0 the input vector $\mathbf{x}_1$ was not in the set of inputs able to produce $\mathbf{q}_0$. If period t technology had been available then since $\mathbf{x}_t$ is in $V_t(\mathbf{q}_0)$ a cost-minimising firm could have adopted it at a cost of $\mathbf{w}_0'\mathbf{x}_t$. Given this violation of WACM, under our assumptions we know that, in order to correct this we need to set $\mathbf{x}_t^*$ such that

$$\begin{array}{rcl}\mathbf{w}_0'\mathbf{x}_0 & \leq & \mathbf{w}_0'\mathbf{x}_t^* \\ \mathbf{x}_t & \leq & \mathbf{x}_t^*\end{array}$$

That is, we need to scale up the apparent usage of inputs in period t. However, not all violations of the conditions in Theorem 1 are reconcilable with our assumptions. In particular consider another observation

$$\begin{array}{rcl}\mathbf{q}_t & \leq & \mathbf{q}_0 \\ \mathbf{w}_t'\mathbf{x}_t & > & \mathbf{w}_t'\mathbf{x}_0\end{array}$$

We then have the next result which relates this other type of observation to our economic assumptions.

> **Lemma 2**: *The observations* $\mathbf{q}_t \leq \mathbf{q}_0, \mathbf{w}_t'\mathbf{x}_t > \mathbf{w}_t'\mathbf{x}_0$ *and assumptions A1 and A2 are inconsistent.*
>
> **Proof**. Nestedness implies that since $\mathbf{x}_0 \in V_0(\mathbf{q}_0)$ and $\mathbf{q}_0 \geq \mathbf{q}_t$, then $\mathbf{x}_0 \in V_0(\mathbf{q}_t)$. Cost minimisation implies that $\mathbf{x}_0 \notin V_t(\mathbf{q}_t)$. By the definition of productivity improvement $V_0(\mathbf{q}_t) \subseteq V_t(\mathbf{q}_t)$ and hence $\mathbf{x}_0 \in V_t(\mathbf{q}_t)$ which is a contradiction.■

Lemma 2 shows that not all WACM violations can be rationalised with our assumptions and hence this gives a condition which can be used to test our identifying assumptions. In particular if we see output dropping or constant over time, whilst there are increases in costs, then this is impossible to reconcile with the assumptions of cost minimising behaviour and technical improvement.

To summarise, if we have a dataset $\{\mathbf{q}_t, \mathbf{w}_t, \mathbf{x}_t\}_{t=1,...,T}$ which exhibits Lemma 2-type violations of WACM then such data are fundamentally inconsistent with these assumptions. If it contains only Lemma 1-type violations, then there exist and we can seek, suitable adjustments to the input data such that they satisfy WACM and the assumed direction of technical change. These adjusted input data can then be used to compute a bound on an index of productivity improvement. We now address the question of how we might do this. Consider the following distance measure

$$\Delta_t\left(V_0, \mathbf{x}_t\right) = \min_{\delta_t} \left\{ \delta_t : \begin{array}{c} \text{Statement (1) in Theorem (1)} \\ \text{holds for } \{\mathbf{w}_t, \mathbf{q}_t, \mathbf{x}_t^*\} \\ \mathbf{x}_t^* = \delta_t \mathbf{x}_t,\ \delta_t > 0 \end{array} \right\}$$

for $t = 0, 1, ..., T$. The distance δ_t which solves this problem is the smallest radial distance such that there exists a family of nontrivial, closed, convex, positive monotonic input requirement sets $\{V_0(\mathbf{q})\}$ such that the data $\{\mathbf{w}_t, \mathbf{q}_t, \mathbf{x}_t^*\}$ solves the problem $\min_{\mathbf{x}^*} \mathbf{w}_t'\mathbf{x}^*$ subject to $\mathbf{x}^*$ is in $V_0(\mathbf{q}_t)$ for each $t = 1, ..., T$. That is, δ_t is the minimum amount, over all reference technologies, by which the observed inputs $\mathbf{x}_t$ need to be scaled up in order to continue to produce current outputs using reference period technology. Any specific choice of functional form for firm technology will result in a greater distance. That is:

Theorem 2: $\Delta_t(V_0, \mathbf{x}_t) \leq D_t(V_0, \mathbf{x}_t)$ *for all* $D_t(V_0, \mathbf{x}_t)$.

Proof. $\Delta_t(V_0, \mathbf{x}_t) > D_t(V_0, \mathbf{x}_t)$ violates the definition of $\Delta_t(V_0, \mathbf{x}_t)$. ■

Hence if we can find $\Delta_t(V_0, \mathbf{x}_t)$ we can find a lower bound on our desired productivity measure, that is

$$\frac{\Delta_t(V_0, \mathbf{x}_t)}{\Delta_t(V_t, \mathbf{x}_t)} \leq \frac{D_t(V_0, \mathbf{x}_t)}{D_t(V_t, \mathbf{x}_t)}$$

since the denominators are equal. The problem of how to calculate $\Delta_t(V_0, \mathbf{x}_t)$ is a simple quadratic programming problem

$$\min_{\delta_t} \sum_{t=1}^{T} \sum_{j=1}^{J} \left(x_t^j - x_t^{j*}\right)^2$$

$$\begin{aligned} \text{subject to } \mathbf{w}_i'\mathbf{x}_j^* &\geq \mathbf{w}_i'\mathbf{x}_i^* \text{ for } \mathbf{q}_j \geq \mathbf{q}_i \\ \text{and } \mathbf{x}_t &\leq \mathbf{x}_t^* \\ \text{and } \mathbf{x}_t^* &= \delta_t \mathbf{x}_t \\ \text{and } \boldsymbol{\delta}_{t-1} &\leq \delta_t \end{aligned}$$

which is a modification of the approach used in Varian (1985) to adapt tests of WACM to situations in which the data are measured with error. Let X denote the set of all input vectors which satisfy WACM given output and price data $\{\mathbf{q}_t, \mathbf{w}_t\}_{t=1,...,T}$, that is

$$X = \{\mathbf{x}_t^* : \mathbf{x}_t^* \text{ satisfies WACM given } \{\mathbf{q}_t, \mathbf{w}_t\}, t = 1, ..., T\}$$

Then δ_t is the distance between an observed input vector $\mathbf{x}_t$ and this set. Note the following result:

Theorem 3: *X is non-empty and is convex.*

Proof. To see that X non-empty note that a collection of T identical but otherwise arbitrary vectors of length J will always satisfy WACM for a given set of outputs and input prices $\{\mathbf{q}_t, \mathbf{w}_t\}_{t=1,...,T}$. Consider two datasets $\{\mathbf{q}_t, \mathbf{w}_t, \widehat{\mathbf{x}}_t\}_{t=1,...,T}$ and $\{\mathbf{q}_t, \mathbf{w}_t, \widetilde{\mathbf{x}}_t\}_{t=1,...,T}$ and suppose that they satisfy WACM. Therefore $\{\widehat{\mathbf{x}}_t\}_{t=1,...,T} \in X$ and $\{\widetilde{\mathbf{x}}_t\}_{t=1,...,T} \in X$. We need to show that $\{\mathbf{x}_t\}_{t=1,...,T} \in X$ where $\mathbf{x}_t = \lambda\widehat{\mathbf{x}}_t + (1-\lambda)\widetilde{\mathbf{x}}_t$, $\lambda \in [0,1]$. Since both datasets pass WACM we have $\mathbf{q}_j \geq \mathbf{q}_i \Rightarrow \mathbf{w}_i'\widehat{\mathbf{x}}_j \geq \mathbf{w}_i'\widehat{\mathbf{x}}_i$ and $\mathbf{w}_i'\widetilde{\mathbf{x}}_j \geq \mathbf{w}_i'\widetilde{\mathbf{x}}_i$ for all i, j. Note that $\mathbf{w}_i'\widehat{\mathbf{x}}_j \geq \mathbf{w}_i'\widehat{\mathbf{x}}_i \Rightarrow \mathbf{w}_i'[\lambda\widehat{\mathbf{x}}_j] \geq \mathbf{w}_i'[\lambda\widehat{\mathbf{x}}_i]$ and $\mathbf{w}_i'\widetilde{\mathbf{x}}_j \geq \mathbf{w}_i'\widetilde{\mathbf{x}}_i \Rightarrow \mathbf{w}_i'[(1-\lambda)\widetilde{\mathbf{x}}_j] \geq \mathbf{w}_i'[(1-\lambda)\widetilde{\mathbf{x}}_i]$. Summing across these inequalities gives

$$\begin{aligned}
\mathbf{w}_i'[\lambda\widehat{\mathbf{x}}_j] + \mathbf{w}_i'[(1-\lambda)\widetilde{\mathbf{x}}_j] &\geq \mathbf{w}_i'[\lambda\widehat{\mathbf{x}}_i] + \mathbf{w}_i'[(1-\lambda)\widetilde{\mathbf{x}}_i] \\
\mathbf{w}_i'[\lambda\widehat{\mathbf{x}}_j + (1-\lambda)\widetilde{\mathbf{x}}_j] &\geq \mathbf{w}_i'[\lambda\widehat{\mathbf{x}}_i + (1-\lambda)\widetilde{\mathbf{x}}_i] \\
\mathbf{w}_i'\mathbf{x}_j &\geq \mathbf{w}_i'\mathbf{x}_i
\end{aligned}$$

Hence $\mathbf{q}_j \geq \mathbf{q}_i \Rightarrow \mathbf{w}_i'\mathbf{x}_j \geq \mathbf{w}_i'\mathbf{x}_i$ and therefore the data $\{\mathbf{q}_t, \mathbf{w}_t, \mathbf{x}_t\}_{t=1,...,T}$ satisfy WACM and $\{\mathbf{x}_t\}_{t=1,...,T} \in X$.■

As a result, since X is a convex set, the numerical computations involved in solving for δ_t are straightforward and, at least for reasonably sized datasets, are unlikely to be time-consuming.

To sum up: what we are suggesting is the calculation of the minimum bound on a productivity index for the firm for all production technologies which are nonparametrically consistent with the Weak Axiom of Cost Minimisation and technological change in a known direction. There is a method of calculating this bound in finite datasets which we show to be computationally straightforward. In practice in a sample of firms this approach can be adopted firm-by-firm allowing for complete heterogeneity in production technologies and productivity changes across firms which is unrestricted other than by the two identifying assumptions.

3 An Empirical Illustration

We now apply the ideas to measure the productivity in a sample of Danish farms. The sample consists of farms that produce beef, crops and milk. Some of these farms probably have improved the productivity during the period studied which is 1980 to 1991 for two reasons. The first relates to the fact that new pesticides were taken into use during the 1980's which were more efficient fighting crop diseases especially on winter crops and which thus made winter crops more profitable. The second relates to the adoption of manure tanks amongst animal husbandry producers. This enabled a more efficient use of the manure, since it became possible for farmers to store the manure until it was needed on the fields as an input in crop production.

The productivity indices calculated in this paper use data provided by the Danish Farm Association Service (DFAS) and are unbalanced panel data covering the period 1980 to 1991 with, on average, 1350 farms represented each year. Data are gathered through a voluntary programme involving an intensive consultancy scheme run by DFAS to help the participating farmers increase the profitability of their operations. All participating farmers are asked to complete accounts for the operation of each their production lines (milk, beef, veal, wheat, barley etc.). Thus, for each farm the data includes detailed annual accounts of variable costs and earnings for each production line with corresponding accounts of the quantitative flows of most inputs and outputs. For labour and capital utilisation annual information is available at the farm level but not at the production line level. The participating farmers *a priori* are motivated and have an incentive to provide data of high quality. However, the sample is *not* representative of the Danish population of farmers.

From the farm data we select milk producing farms. Most of these farms also produce other cattle products and crops, but not pork or chicken. The selected sample consist of 1032 farms. Each farm is observed for between 2 and 12 years (see Table 1 for the structure of the panel). Milk producers are restricted by individual milk quotas and cannot maximise profits in a conventional way. Nevertheless, cost minimising behaviour may be a realistic assumption. The main inputs in milk, beef and crop production are: fodder, cattle, nitrate (from fertiliser and manure), pesticides, and the services from labour, land, building and machine capital[4]. Since a cow usually is used for milk production in several periods (on average 2.5 years) we have calculated user costs for each cow, which reflect the prices of the cows' services. For pesticides the costs are observed but not the applied quantities. Average yearly pesticide prices have been collected from Statistics Denmark and used to calculate the quantities applied.

We test each individual member of the farm sample for violations of WACM and report the results in Table 1. The column labelled "Farms" reports the number of farms in the sample, by the number of years they remain in the panel. The "Lemma 1" column reports the number of farms with at least one lemma 1 violation of WACM[5] and no lemma 2 violations[6] and the "Lemma 2" column reports the number of farms with at least one lemma 2 violation. All

[4] In what follows capital and land are assumed to be fixed inputs.

[5] Recall that a violation of this type occurs when outputs grow over time, and costs (at initial period prices) are falling.

[6] Recall that a violation of this type occurs when outputs fall over time, and costs (at current prices) are rising. The type of violation is inconsistent with our approach.

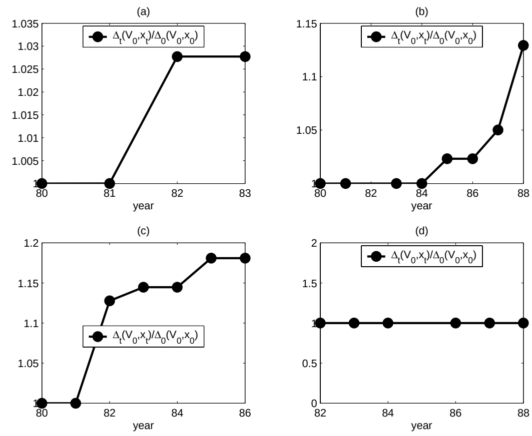
the results are reported by the number of years the farms are observed. The "Farms" column also reports the column percentages – e.g. 263 farms or 25% of the sample, are observed for 2 years – while the "Lemma 1" and "Lemma 2" columns also report the row percentages – e.g. of the 203 farms observed for 3 years, 13 (or 6%) exhibit lemma 1 violations, and 7 (or 3%) exhibit lemma 2 violations.

Table 1: Number of Farms, Lemma 1 and Lemma 2 Errors by the number of Years the Farms are in the Panel

No of years	Farms		Lemma 1		Lemma 2	
2	263	25%	4	2%	4	2%
3	203	20%	13	6%	7	3%
4	172	17%	11	6%	10	6%
5	111	11%	18	16%	10	9%
6	110	11%	27	25%	13	12%
7	73	7%	20	27%	6	8%
8	31	3%	8	26%	7	23%
9	37	4%	13	35%	5	14%
10	17	2%	4	24%	3	18%
11	10	1%	2	20%	2	20%
12	5	0%	1	20%	1	20%
Total	1032	100%	121	12%	68	7%

From Table 1 we see that 68 farms (7% of the sample) have at least one lemma 2 violation of WACM and are not consistent with the identifying assumptions of cost minimising behaviour and productivity improvement. We also see that 121 farms (12% of the sample) have lemma 1 violations of WACM and thus are consistent with an improved production technology. The rest (1032-68-121=883 farms) are consistent with WACM and it cannot be rejected that these farms have an intertemporally stable production technology. There is therefore no evidence at all that these farms have experienced any productivity changes. The share of farms with lemma 1 violations of WACM increases with the number of years the farms are observed. Naturally, this may be because productivity increases are more likely to show over a longer time span. For the 121 farms with lemma 1 violations we individually solve the quadratic programming problem. Note that for 2 farms we cannot find a solution because of apparent numerical problems which we have been unable to diagnose and we set them aside for the moment. This leaves 119 farms for which our numerical procedures run smoothly. From the perturbed data and the observed data we calculate Malmquist quantity indices that measure the lower bound on the productivity improvement for each farm. For the 883 farms that satisfy WACM the lower bound on the productivity index is 1 in all the periods - corresponding to a constant productivity. Figure 1 shows four examples of the productivity indices calculated for selected farms. These show the typical sorts of patterns we are recovering from the data.

FIGURE 1: Examples of Productivity Indices



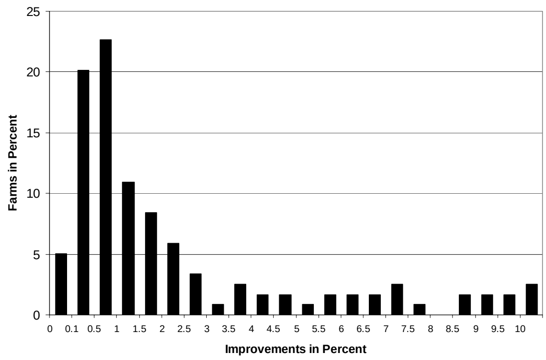
Years are plotted along the horizontal axis, while the Malmquist productivity index numbers, $\Delta_t(V_0, \mathbf{x}_t)/\Delta_0(V_0, \mathbf{x}_0)$, are plotted along the vertical axis. An observation is marked with a circle. Since $\Delta_0(V_0, \mathbf{x}_0) = 1$, $\Delta_t(V_0, \mathbf{x}_t)/\Delta_0(V_0, \mathbf{x}_0) = \Delta_t(V_0, \mathbf{x}_t)$,which is to the smallest radial adjustment of the costs that ensures the data satisfy WACM. Figure 1a shows the productivity index for a firm which is observed for 4 years in the period 1980 to 1983. From 1981 to 1982 the input productivity did increase by at least 2.5 percent, which is also the total productivity increase for the entire observed period. Figure 1b and 1c show productivity indices respectively for a farm that is observed for 8 years in the period 1980 to 1988 and a farm that is observed for 7 years in the period 1980 to 1986. For both farms the calculated productivity indices increases in three steps. The horizontal productivity index in figure 1d shows the lower bound of the productivity index for the 883 farms that satisfy WACM and that are consistent with an intertemporally stable production technology.

For each of the 119 farms with productivity improvements we calculate the average productivity growth rate per year and we group the farms by the average growth rate. The distribution of average growth rates are reported in Figure 2 (and Table 2). The horizontal axis shows the grouped average productivity growth rates while the vertical axis shows the share of farms in the specified groups. From the left, the first bar reports that about 5% of the 119 farms have average growth rates larger than 0% and smaller than 0.1%. Similarly, the second bar shows that 21% of the farms have average growth rates between 0.1% and 0.5%. Most of the average growth rates are relatively small with 80% of the farms have average growth rates less than 4% percent. The new pesticide technologies and the manure tanks mainly affect the input productivity in crop production. The variable costs for producing milk and beef may only be affected to small extent by the productivity increase. Thus, small average growth rates seem likely.

Table 2: Average Yearly Minimum Productivity Improvement

Improvement	N	Percentage	Cumulative Percentage
$0 - 0.1\%$	6	5	5
$0.1 - 0.5\%$	25	21	26
$0.5 - 1\%$	27	23	49
$1 - 1.5\%$	13	11	60
$1.5 - 2\%$	10	8	68
$2 - 3\%$	11	9	77
$3 - 4\%$	4	3	80
$4 - 5\%$	4	3	83
$5 - 6\%$	3	3	86
$6 - 7\%$	4	3	89
$7 - 8\%$	4	3	92
$8 - 9\%$	2	2	94
$9 - 10\%$	4	3	97
$10 - 34\%$	3	3	100
Total	119		

Figure 2: The distribution of productivity changes (non-zero values)



4 Conclusions

In this paper we have shown how the evolution of a firm's productivity over time may be nonparametrically inferred from data on its inputs, outputs and their corresponding prices over time. We have shown how this can be made possible by making two identifying assumptions. One assumption is on firm behaviour, the other is on the direction of productivity change. We have also shown how these identifying assumptions are empirically rejectable. Where they are not rejected we propose a nonparametric method of calculating the minimum rate of productivity improvement for each firm based on the distance function which is computationally straightforward. Finally, we have illustrated these ideas using a sample of Danish farms.

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